Partial response signalling

Remember intersymbol interference?

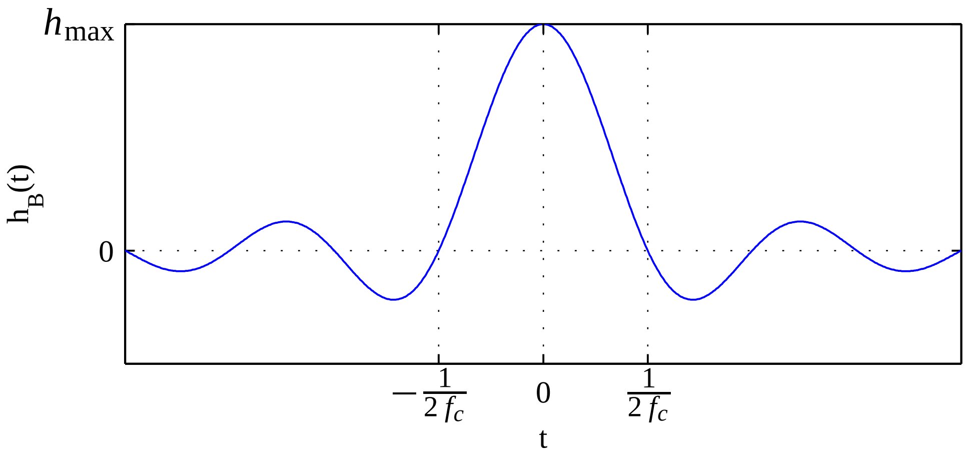
A time-limited pulse, which is useful for signalling, occupies an infinite frequency range. A frequency-limited pulse has an infinite time duration. It seems therefore that we either have to settle for interchannel interference (in frequency) or intersymbol interference (in time).

This is not the case, though — consider the ideal ("brickwall") lowpass filter

$$H_B(f) = \begin{cases} 1 & |f| < f_c \\ 0 & \text{elsewhere} \end{cases}$$

with impulse response

$$h_B(t) = 2f_c \frac{\sin(2\pi f_c t)}{2\pi f_c t}$$



This is bandlimited, but *also* exhibits no intersymbol interference (ISI) as long as the response is sampled only at integer multiples of $1/(2f_c)$. Thus, using infinitely sharp filters it is possible to transmit $2f_c$ symbols per second without ISI.

There are problems with the use of this filter:

- It is unrealisable, and difficult to approximate

- The amplitude falls off linearly with time, making the system sensitive to timing errors.

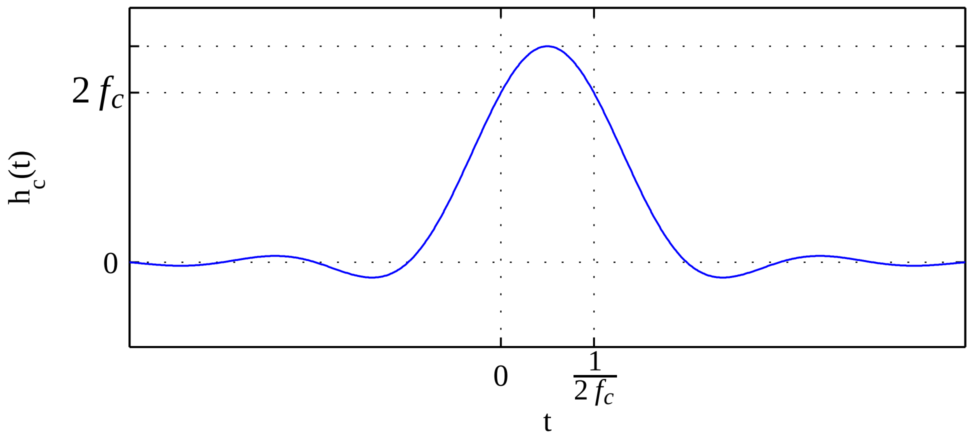
In **partial response signalling**, also called duobinary signalling or correlative coding, the idea is to introduce a controlled amount of ISI into the signal rather than trying to eliminate it completely. This can be compensated for at the receiver, thereby achieving the ideal symbol-rate packing of 2 symbols per Hertz, but without the requirements of unrealisable filters.

Consider the cosine filter with transfer function

$$H_c(f) = \begin{cases} 2e^{-\frac{j\pi f}{2f_c}} \cos\left(\frac{\pi f}{2f_c}\right) & |f| < f_c \\ 0 & \text{otherwise} \end{cases}$$
$$= H_B(f)\left[2e^{-\frac{j\pi f}{2f_c}} \cos\left(\frac{\pi f}{2f_c}\right)\right] = H_B(f)\left[1 + e^{-\frac{j\pi f}{f_c}}\right],$$

and corresponding impulse response

$$h_c(t) = h_B(t) + h_B(t - T_b) = \frac{8f_c}{\pi}\left[\frac{\cos\left[2\pi f_c\left(t - \frac{1}{4f_c}\right)\right]}{1 - (4f_c)^2\left(t - \frac{1}{4f_c}\right)^2}\right].$$



This is better than the brickwall filter:

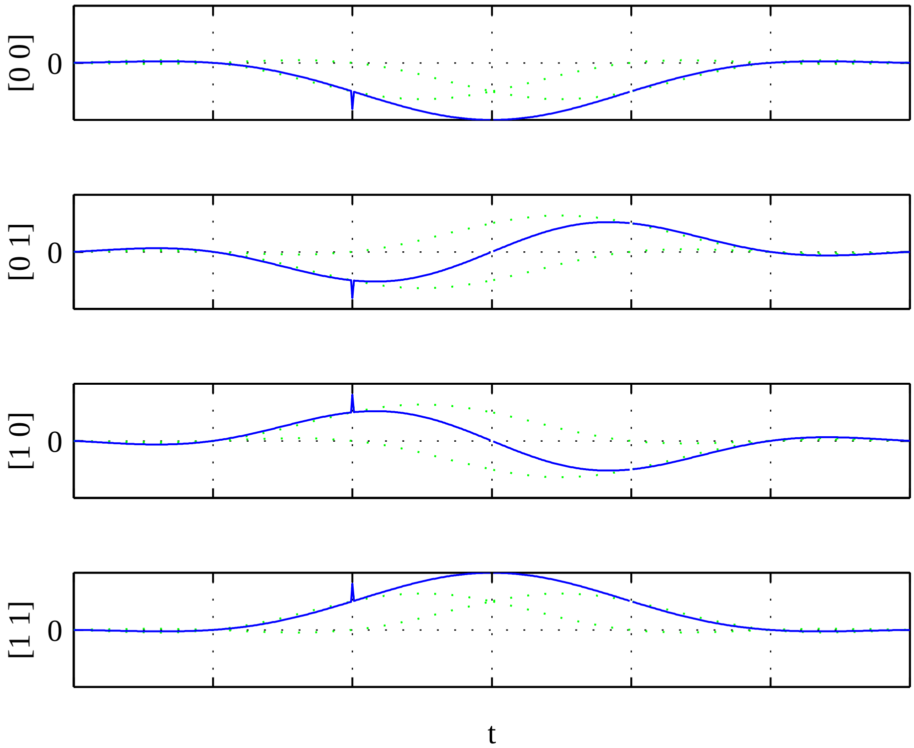
- It is still unrealisable, but easier to approximate

- The sidelobes fall of as t^2.

However, the central peak is now has width $\frac{3}{2f_c}$, whereas for the ideal filter it was $\frac{2}{2f_c}$.

Suppose we use the given pulse for signalling. Since the pulse is now nonzero at *two* consecutive time instants, there will clearly be intersymbol interference at the output. However, the amount of interference is known if we know the pulse that was received at the previous time instant.

For example, consider a polar signal representation scheme.



If the pulse received at time $t = 0$ was a one, then we know that we can expect an ISI contribution of T_b at $t = T_b$. When we sample the received signal at $t = T_b$, we know that we must just allow for this amount to effectively eliminate the ISI.

Example:

Suppose the sequence 0010110 is transmitted, with the first bit assumed to be a startup digit, not part of the data. Then

Digit x_k	0	0	1	0	1	1	0
Bipolar amplitude	-1	-1	1	-1	1	1	-1
Combined amplitude		-2	0	0	0	2	0

The decision rule for decoding is as follows:

$$y_k = 2 \qquad \text{(decide } x_k = 1\text{)}$$
$$y_k = -2 \qquad \text{(decide } x_k = 0\text{)}$$
$$y_k = 0 \qquad \text{(decide opposite of previous decision)}$$

Using this rule gives

Decoded values	-1	1	-1	1	1	-1
Decoded sequence	0	1	0	1	1	0

A problem with the system as described is that errors will tend to propagate if at any stage a digit is decoded erroneously. This can be avoided by precoding the input data stream x_k to form the output b_k according to the relation

$$b_k = x_k \oplus b_{k-1}.$$

The decision rule upon observing the sample y_k then becomes

$$y_k = \pm 2 \qquad \text{(decide } x_k = 0\text{)}$$
$$y_k = 0 \qquad \text{(decide } x_k = 1\text{)}.$$

Example: repeating the previous example with precoding,

Digit x_k	0	0	1	0	1	1	0
Precoded sequence	0	0	1	1	0	1	1
Bipolar amplitude	-1	-1	1	1	-1	1	1
Combined amplitude		-2	0	2	0	0	2
Decoded sequence		0	1	0	1	1	0