EEE482F: Problem Set 1

1. A digital source emits -1.0 and 0.0V levels with a probability of 0.2 each, and $+3.0$ and $+4.0$V levels with a probability of 0.3 each. Evaluate the average information of the source.

2. Consider a source that produces 6 messages with probabilities $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, and $\frac{1}{32}$. Determine the average information content of a message.

3. A given source alphabet consists of 300 words, of which 15 occur with probability 0.06 each and the remaining 285 words occur with probability 0.00035 each. If 1000 words are transmitted each second, what is the average rate of information transmission.

4. A numeric keypad has the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Assume that the probability of sending any one digit is the same as that for sending any of the other digits. Calculate how often the buttons must be pressed in order to send out information at a rate of 2 bits/second.

5. Consider a voice-grade telephone circuit with a bandwidth of 3kHz. Assume that the circuit can be modelled as an additive white Gaussian noise (AWGN) channel.
   (a) What is the capacity of such a circuit if the SNR is 30dB.
   (b) What is the minimum SNR required for a data rate of 4800 bits/s on such a voice grade circuit?
   (c) Repeat part (b) for a data rate of 19200 bits/s.

6. A 100 kbit/s data stream is to be transmitted on a voice-grade telephone circuit with a bandwidth of 3kHz. Is it possible to achieve error-free transmission with a SNR of 10dB?

7. Answer the following:
   (a) Find the average capacity in bits per second that would be required to transmit a high-resolution black-and-white TV signal at the rate of 32 pictures per second if each picture is made up of 2×10^6 picture elements (pixels) and 16 different brightness levels. All pixels are assumed to be independent and all levels have equal likelihood of occurrence.
   (b) For colour TV, this system additionally provides for 64 different shades of colour. How much more system capacity is required for a colour system compared to the black-and-white system?
   (c) Find the required capacity if 100 of the possible brightness-colour combinations occur with a probability of 0.003 each, 300 of the combinations occur with a probability of 0.001, and 624 of the combinations occur with a probability of 0.00064.

8. Assume that a computer terminal has 110 characters on its keyboard and that each character is sent using binary words.
   (a) What are the number of bits required to represent each character?
   (b) How fast can characters be sent (characters/sec) over a telephone line channel having a bandwidth of 3.2kHz and a signal-to-noise ratio of 20dB?
   (c) What is the entropy of each character if each is equally likely to be sent?

9. In a binary PCM system, if the quantising noise is not to exceed $\pm P$ percent of the peak-to-peak analogue level, show that the number of bits in each PCM word needs to be

$$n \geq (\log_2 10) \log_{10}\left(\frac{50}{P}\right) = 3.32 \log_{10}\left(\frac{50}{P}\right).$$

10. The information in an analogue voltage waveform is to be transmitted over a PCM system with a $\pm 0.1\%$ accuracy (full scale). The analogue waveform has an absolute bandwidth of 100Hz and an amplitude range of -10 to $+10$V.
   (a) Determine the minimum sampling rate needed.

(b) Determine the number of bits needed in each PCM word.

(c) Determine the minimum bit rate required in the PCM channel.

(d) Determine the minimum absolute channel bandwidth required for transmission of this PCM signal.

11. A 40-Mbyte hard disk is used to store PCM data. Suppose that a VF (voice-frequency) signal is sampled at 8 ksamples/sec and the encoded PCM is to have an average S/N of at least 30dB. How many minutes of VF conversation (i.e. PCM data) can be stored on the hard disk?

12. Given an audio signal with spectral components in the frequency band 300 to 3000Hz, assume that a sampling rate of 7kHz will be used to generate a PCM signal. Design an appropriate PCM system, as follows:

(a) Draw a block diagram of the PCM system including the transmitter, channel, and receiver.

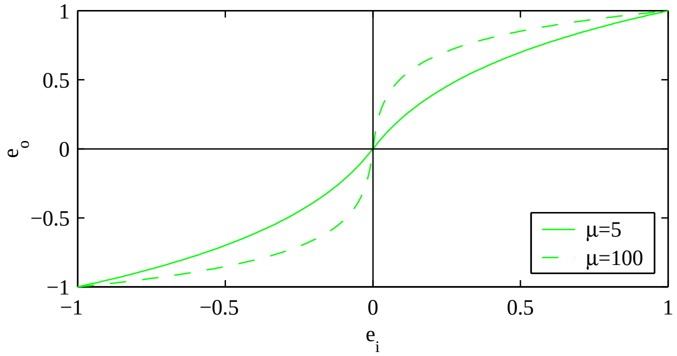
(b) Specify the number of uniform quantisation levels needed and the zero-crossing channel bandwidth required, assuming that the peak signal-to-noise ratio at the receiver output needs to be at least 30dB and that polar NRZ signalling is used.

(c) Discuss how nonuniform quantisation can be used to improve performance.

13. A common compression characteristic using in companding is the μ-law characteristic

$$e_o = \text{sign}(e_i)\frac{\ln(1+\mu|e_i|)}{\ln(1+\mu)},$$

which is normalised for compression over the range -1 to 1. A plot of the characteristic is shown below:



(a) Sketch the complete $\mu = 10$ characteristic that will handle input voltages over the range -5V to $+5$V.

(b) Plot the corresponding expander characteristic.

(c) Draw a 16-level nonuniform quantiser characteristic that corresponds to the $\mu = 10$ compression characteristic.

14. For a 4-bit PCM system, calculate and sketch a plot of the output S/N (in decibels) as a function of the relative input level, $20\log(x_{\text{rms}}/V)$ for

(a) A PCM system that uses $\mu = 10$ law companding.

(b) A PCM system that uses uniform quantisation (no companding).

Assume that a triangular-type waveform is present at the input so that $< |x| >= \sqrt{3}x_{\text{rms}}/2$. Which of these systems is better to use in practice? Why?

15. Suppose 100 voltage levels are employed to transmit 100 equally likely messages. Assume $\lambda = 3.5$ and the system bandwidth $B = 10^4$Hz.

(a) Calculate S/η.

(b) If an integrate-and-dump filter is employed to determine which level is sent, calculate the probability of an error when sending the kth level. Assume that the only errors possible are in choosing the $k-1$ or the $k+1$ levels.

16. Consider a baseband unipolar communication system with equally likely signalling:

$$s(t) = \begin{cases} +A, & 0 < t \le T \quad \text{(binary 1)} \\ 0, & 0 < t \le T \quad \text{(binary 0)} \end{cases}$$

Assume that the receiver uses a simple RC LPF with a time constant of $RC = \tau$ where $\tau = T$ and $1/T$ is the bit rate. (By "simple" it is meant that the initial conditions of the LPF are *not* reset to zero at the beginning of each bit interval.)

   (a) For signal alone at the receiver input, evaluate the approximate worst-case signal to ISI ratio (in decibels) out of the LPF at the sampling time $t = t_0 = nT$, where n is an integer.

   (b) Evaluate the signal to ISI ratio (in decibels) as a function of the parameter K, where $t = t_0 = (n + K)T$ and $0 < K \le 1$.

   (c) What is the optimum sampling time to use to maximise the signal-to-ISI power ratio out of the LPF?

17. For unipolar baseband signalling with pulses

$$s(t) = \begin{cases} +A, & 0 < t \le T \quad \text{(binary 1)} \\ 0, & 0 < t \le T \quad \text{(binary 0)}, \end{cases}$$

   (a) Find the matched-filter frequency response and show how the filtering operation can be implemented by using an integrate-and-dump filter.

   (b) Show that the equivalent bandwidth of the matched filter is $B_{\text{eq}} = 1/(2 * T) = R/2$.

18. Equally likely polar signalling is used in an baseband communication system. Gaussian noise having a PSD of $N_0/2$ W/Hz plus a polar signal with a peak level of A volts is present at the receiver input. The receiver uses a matched-filter circuit having a voltage gain of 1000.

   (a) Find the expression for P_ϵ as a function of A, N_0, T, and V_T, where $R = 1/T$ is the bit rate and V_T is the threshold level.

   (b) Plot P_ϵ as a function of V_T for the case of $A = 8 \times 10^{-3}$ V, $N_0/2 = 4 \times 10^{-9}$ W/Hz, and $R = 1200$ bits/sec.

19. Assume a typical binary sequence and show that if the corresponding polar NRZ signal and unipolar NRZ signal have the same peak-to-peak amplitude, the polar signal has less power (an advantage) than the unipolar signal. If noise is added to these signals, how do the probabilities of bit errors compare for the two signalling techniques.

20. A binary communication system uses polar signalling. The overall impulse response is designed to be of the $(\sin x)/x$ type so that there will be no ISI. The bit rate is $R = f_s = 300$ bits/sec.

   (a) What is the bandwidth of the polar signal?

   (b) Plot the waveform of the polar signal at the system output when the input binary data is 01100101. Can you discern the data by looking at this polar waveform?

21. An analogue signal is to be converted into a polar PCM signal and transmitted over a channel that is absolutely bandlimited to 4kHz. Assume that 16 quantisation levels are used and that the overall equivalent system transfer function is of the raised cosine-rolloff type with $r = 0.5$.

   (a) Find the maximum PCM bit rate that can be supported by this system without introducing ISI.

   (b) Find the maximum bandwidth that can be permitted for the analogue signal.

22. A 1 bit/sec unipolar NRZ signal (rectangular bit shape) is sent over a bandlimited channel. The channel has the impulse response $h_c(t) = e^{-1000t}u(t)$. Design a 5-tap zero-forcing transversal filter to equalise the channel response. Assume that $h_c(t)$ has significant values only over the first three bit intervals.

23. A 1 bit/sec binary signal is to be transmitted over a channel. The

equivalent impulse response is

$$h_e(t) = \begin{cases} e^{-t}, & t \geq 0 \\ e^{-t^2}, & t < 0. \end{cases}$$

(a) Plot the impulse response.

(b) Design a transversal filter to force four points (at the sampling times) to zero.

(c) Plot the impulse response that includes the zero-forcing equalising filter.

24. A DM system is tested with a 10kHz sinusoidal signal, 1V peak-to-peak, at the input. It is sampled at 10 times the Nyquist rate.

(a) What is the step size required to prevent slope overload and to minimise granular noise?

(b) What is the power of the granular noise?

(c) If the receiver input is bandlimited to 200kHz, what is the average-signal/quantising-noise power ratio?

25. Assume that the input to a DM is $0.1t^8 - 5t + 2$. The step size of the DM is 1V, and the sampler operates at 10 samples/sec. Over a time interval of 0 to 2 sec, sketch the input waveform, the delta modulator output, and the integrator output. Denote the granular noise and the slope overload regions.

26. A delta modulator is to be designed to transmit the information of an analogue waveform that has a peak-to-peak level of 1V and a bandwidth of 3.4kHz. Assume that the waveform is to be transmitted over a channel where the frequency response is extremely poor above 1MHz.

(a) Select the appropriate step size and sampling rate for a sine-wave test signal and discuss the performance of the system using the parameter values you have selected.

(b) If the DM system is to be used to transmit the information of a voice (analogue) signal, select the appropriate step size when the sampling rate is 25kHz. Discuss the performance of the system under these conditions.