M-ary PSK

Multi-level modulation techniques permit high data rates within fixed bandwidth constraints. A convenient set of signals for M-ary PSK is

$$\phi_i(t) = A\cos(\omega_c t + \theta_i), \qquad 0 < t \leq T_s,$$

where the M phase angles are

$$\theta_i = 0, \frac{2\pi}{M}, \ldots, \frac{2(M-1)\pi}{M}.$$

For equiprobable ones and zeros the PSD for M-ary PSK is

$$S_\phi(\omega) = A^2 T_s \mathrm{Sa}^2\left[(\omega - \omega_c)\frac{T_s}{2}\right].$$

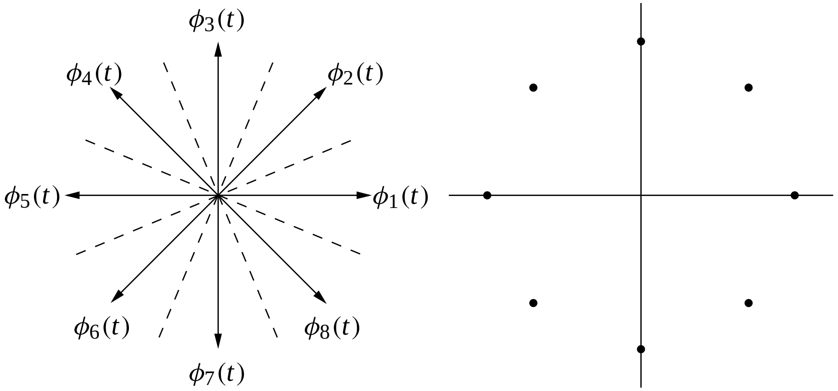
The symbols in this case are of duration T_s, so the information (or bit) rate T_b satisfies

$$T_s = T_b \log_2 M.$$

The potential bandwidth efficiency of M-ary PSK can be shown to be

$$\frac{f_b}{B} = \log_2 M \text{ bps/Hz}.$$

A phase diagram and signal constellation diagram for the case of $M = 8$ are shown below:



All signals have the same energy E_s over the interval $(0, T_s)$, and each signal is correctly demodulated at the receiver if the phase is within $\pm\pi/M$ of the correct phase θ_i. No information is contained in the energy of the signal.

A probability of error calculation involves analysing the received phase at the receiver (in the presence of noise), and comparing it to the actual phases. An exact solution is difficult to compute, but for $P_\epsilon < 10^{-3}$ an approximate probability of making a symbol error is

$$P_\epsilon \approx 2\mathrm{erfc}\sqrt{\frac{2E_s}{\eta}\sin^2\frac{\pi}{M}}, \qquad M > 2.$$

If a Gray code is used, then the corresponding bit error is approximately

$$P_{\mathrm{be}} \approx P_\epsilon / \log_2 M.$$

Stremler provides a table of the SNR requirements of M-ary PSK for fixed error rates. The results indicate that for QPSK ($M = 4$) has definite advantages over coherent PSK ($M = 2$) — the bandwidth efficiency is doubled for only about a 0.3dB increase in SNR. For higher-rate transmissions in bandlimited channels the choice $M = 8$ is often used. Values of $M > 8$ are seldom used due to excessive power requirements.

M-ary PSK requires more complex equipment than BPSK signalling. Carrier recovery is also more complicated. The requirement that the carrier be recovered can be mitigated by using a comparison between the phases of two successive symbols. This leads to M-ary *differential* PSK, and is in principle similar to DPSK (which is differential PSK for $M = 2$).

For large SNR the probability of error is

$$P_\epsilon \approx 2\text{erfc}\sqrt{\frac{2E_s}{\eta}\sin^2\frac{\pi}{\sqrt{2M}}}.$$

Thus differential detection increases the power requirements by the factor

$$\Gamma = \frac{\sin^2\pi/M}{\sin^2(\pi/\sqrt{2M})}$$

For $m = 4$, the increase is required power is about 2.5dB, which may be justified by the saving in equipment complexity.