CFSK and MSK

QPSK signalling is efficient from a bandwidth efficiency perspective, achieving 2 bps per Hz of channel bandwidth. However, away from the carrier frequency the PSD falls off as ω^{-2}. This may not be fast enough, particularly in nonlinear channels.

Continuous phase FSK (CPFSK) offers a solution to this problem without any compromise in efficiency. This type of modulation is based on FSK techniques, and a frequency shift is used to convey the information, but the phase is controlled to avoid any discontinuities in the signal. The PSD of CPFSK decreases as ω^{-4} away from the carrier frequency.

A CPFSK signal takes the form

$$\phi(t) = A\cos[\omega_c t + \gamma(t)],$$

with the phase $\gamma(t)$ a continuous function of time. Writing in terms of two frequencies ω_1 and ω_2 for marks and spaces we have

$$\phi(t) = A\cos[\omega_c t \pm \Delta\omega t + \gamma(0)],$$

with $\omega_c = (\omega_1 + \omega_2)/2$ and $\Delta\omega = (\omega_1 - \omega_2)/2$. Thus on the interval $0 < t \leq T_b$ the phase is a linear function of time

$$\gamma(t) = \pm\Delta\omega t + \gamma(0).$$

One form of minimum-shift keying (MSK) results if $\Delta\omega = \pi/2T_b$. This is the minimum frequency spacing between ω_1 and ω_2 that allows the two FSK signals to be orthogonal to each other. This criterion means that the frequency separation between f_1 and f_2 must be such that there is a half cycle difference in one bit interval.

Using this condition gives

$$\gamma(t) = \pm\frac{\pi}{2T_b}t + \gamma(0), \qquad 0 < t \leq T_b.$$

Choosing $\gamma(0) = 0$, the possible values of $\gamma(t)$ for $t > 0$ can be shown as a phase trellis diagram. The phase at multiples of T_b can therefore only take on a discrete set of values. More specifically, over each bit interval the phase of the MSK waveform can only be advanced or retarded by exactly 90° — it ramps up by 90° when a 1 is transmitted, and down by 90° when a zero is transmitted.

Let p_k is a switching function taking on values of ± 1 corresponding to the binary input data in the kth signalling interval. Additionally, let γ_k be the phase at the beginning of the kth interval — this is termed the *excess phase*. A recursive phase constraint can then be formulated, which has to be satisfied for the resulting waveform to be continuous:

$$\gamma_k = \frac{\pi}{2}p_k + \gamma_{k-1}.$$

The MSK signal can be expressed as

$$\begin{aligned}\phi_{\mathrm{MSK}}(t) &= A\cos\left[\omega_c t + \frac{\pi t}{2T_b}p_k + \gamma_k\right] \\ &= A\left[\cos\left(\frac{\pi t}{2T_b}p_k + \gamma_k\right)\cos\omega_c t - \sin\left(\frac{\pi t}{2T_b}p_k + \gamma_k\right)\sin\omega_c t\right].\end{aligned}$$

In this last expression we have an in-phase and a quadrature component. Using trigonometric identities, this can be written as

$$\begin{aligned}\phi_{\mathrm{MSK}}(t) &= A\left[\cos\gamma_k\cos\frac{\pi t}{2T_b}\cos\omega_c t - p_k\cos\gamma_k\sin\frac{\pi t}{2T_b}\sin\omega_c t\right] \\ &= A[I(t)\cos\omega_c t - Q(t)\sin\omega_c t].\end{aligned}$$

In this form MSK can be interpreted as being composed of two quadrature data channels $a_I(t)$ and $a_Q(t)$ in an orthogonal QPSK signalling system

$$\phi_{\mathrm{MSK}}(t) = A\left[a_I(t)\cos\frac{\pi t}{2T_b}\cos\omega_c t - a_q(t)\cos\gamma_k\sin\frac{\pi t}{2T_b}\sin\omega_c t\right].$$

However, in this case a sinusoidal pulse weighting, rather than a rectangular weighting, is used to represent the baseband pulses.

Because MSK signalling is essentially equivalent to orthogonal QPSK, the probability of error is the same. Additionally, the bandwidth efficiency is equal, namely 2 bps/Hz.