Phase-shift keying (PSK)

Both OOK and FSK were shown to result in a probability of error of

$$p_\epsilon = \operatorname{erfc}\sqrt{\frac{E_b}{\eta}},$$

where E_b is the average energy per bit.

Stremler's discussion of why OOK and FSK don't perform.

In the simplest case a binary phase-shift keyed (BPSK) signal takes the form

$$s(t) = m(t)\cos(\omega_c t),$$

where ω_c is the carrier frequency, and $m(t)$ is a polar binary baseband signal taking on the value 1 for a mark and -1 for a space. Because there are only two different signals, and they differ only by a change of sign, this signalling scheme is also called **phase-reversal keying** (PRK). In this case the signals sent are

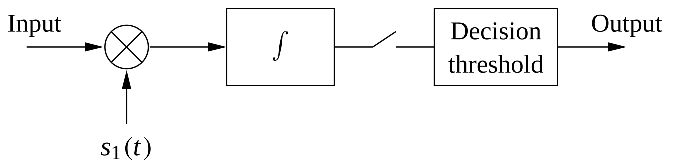
$$s_1(t) = A\cos(\omega_1 t), \qquad 0 < t \leq T,$$
$$s_2(t) = -A\cos(\omega_1 t), \qquad 0 < t \leq T.$$

When the baseband signal consists of square pulses with amplitude taking on values of $+1$ and -1 with equal probability, it can be shown that the power spectral density is

$$P(f) = \frac{1}{4}\left[\frac{\pi T(f - f_c)}{\pi T(f - f_c)}\right]^2 + \frac{1}{4}\left[\frac{\pi T(f + f_c)}{\pi T(f + f_c)}\right]^2.$$

Here the data rate is $R = 1/T$ bits/sec. This is essentially the same as the PSD obtained by assuming a deterministic alternating square wave taking on values of $+1$ and -1 in sequence, which can be considered a worst case.

Coherent or synchronous detection of a PRK signal can be performed by a system of the following form:



Because the signals differ only in sign, there is no need to have two matched filters in the detection. Instead the matched filter nominally takes on a value of E when $s_1(t)$ is present at the input, and $-E$ when $s_2(t)$ is present. The noise variance at the output of the filter is $E\eta/2$, so the overall probability of error at the receiver is

$$P_\epsilon = \operatorname{erfc}\left(\frac{2E}{\sqrt{E\eta/2}}\right) = \operatorname{erfc}\sqrt{\frac{2E}{\eta}}.$$

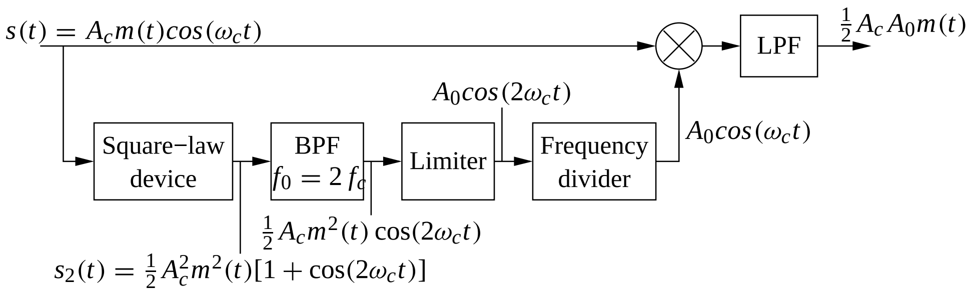
Since the signals have the same energy when a zero and a one are transmitted, this can be written as

$$P_\epsilon = \operatorname{erfc}\sqrt{\frac{2E_b}{\eta}}.$$

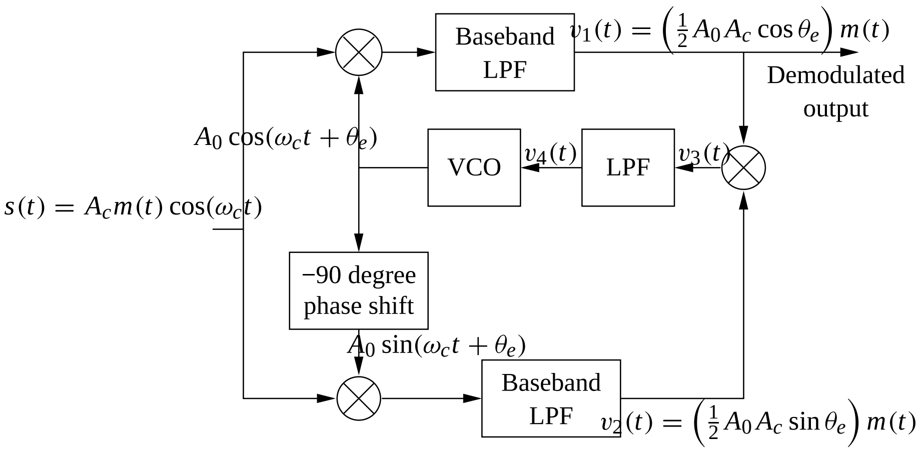
For the same communication channel, PRK therefore requires half the transmission energy for the same bit error probability as OOK and FSK. The bandwidth required is also quite modest when compared with these other signalling schemes.

Note also that the PRK signal can also be obtained in the context of an ASK system modulated by a polar baseband signal. However, there is a 3dB advantage in using PRK over OOK.

A coherent reference for synchronous detection cannot be obtained by the use of an ordinary phase-locked tracking loop, since there are no spectral line components at $\pm f_c$. However, since the signal has a spectrum that is symmetric with respect to the (suppressed) carrier frequency, either a **squaring loop** or a **Costas PLL** can be used to obtain synchronisation. The diagram for a squaring loop in a coherent detector is shown below:



Alternatively, a Costas phase-locked loop can also perform the task:



This can be analysed by assuming that the VCO is locked to the input suppressed carrier frequency f_c, with a constant phase error of θ_e. Then the voltages $v_1(t)$ and $v_2(t)$ are obtained at the output of the baseband lowpass filters as shown. Since θ_e is small, the amplitude of $v_1(t)$ is relatively large compared to that of $v_2(t)$. Furthermore, $v_1(t)$ is proportional to $m(t)$, so it is the demodulated output. The product voltage $v_3(t)$ is

$$v_3(t) = 1/2(1/2 A_0 A_c)^2 m^2(t) \sin 2\theta_e.$$

The voltage $v_3(t)$ is filtered with a LPF that has cutoff frequency near DC so

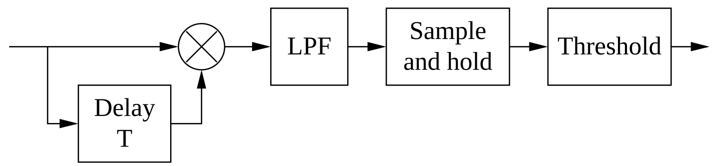
that this filter acts as an integrator to produce the DC VCO control voltage

$$v_4(t) = K \sin 2\theta_e$$

where $K = 1/2(1/2 A_0 A_c)^2 \langle m^2(t) \rangle$ and $\langle m^2(t) \rangle$ is the DC level of $m^2(t)$. This DC control voltage is sufficient to keep the VCO locked to f_c with a small phase error θ_e.

Both of these solutions have one disadvantage — a 180 degree phase ambiguity. It can be shown that the noise performance of the squaring loop and the Costas PLL are equivalent, so the choice of which to implement depends on the relative cost of the loop components and the accuracy that can be realised when each component is built.

Phase-shift keyed signals cannot be detected incoherently. However, a partially coherent technique can be used whereby the phase reference for the present signalling interval is provided by a delayed version of the signal that occurred during the previous sampling interval. A *differential* PSK decoder takes the following form:



If a zero-noise BPSK signal is applied to the receiver input, the output of the sample-and-hold circuit will be positive (binary 1) if the present data bit and the previous data bit are the same; the output is negative (binary zero) if the two data bits are different. Thus if the data in the BPSK is differentially encoded, then the decoded sequence will be recovered at the output of this receiver.

A more general representation for a PSK signal takes the following form:

$$s(t) = A \sin(w_c t + \Delta\theta m(t)).$$

Assume that $m(t)$ has peak values of ± 1. This expression can be expanded as

$$s(t) = A \sin \omega_c t \cos[m(t) \cos^{-1} m] + A \cos \omega_c t \sin[m(t) \cos^{-1} m],$$

where $m = \cos \Delta\theta$ is defined to be the *modulation index*. Recalling that $\cos(x)$ and $\sin(x)$ are even and odd functions of x, this can be written as

$$s(t) = mA \sin \omega_c t + m(t)\sqrt{1 - m^2} A \cos \omega_c t.$$

The first term contains the pilot carrier, while the second carries the data. The average power in the carrier is $m^2 A^2/2$, and the power in the modulation component is $(1 - m^2)A^2/2$. Thus a fraction m^2 of the total power in the modulated waveform is allocated to the carrier. It follows that the carrier component is zero in a PRK waveform, for which $\Delta\theta = \pi/2$.

Stremler indicates that the probability of error for BPSK is

$$P_\epsilon = \text{erfc}\sqrt{2E(1 - m^2)/\eta}.$$

Thus the effect of allocating a fraction m^2 of the total power to the carrier is to degrade P_ϵ by an equivalent S/N loss of $10 \log_{10}(1 - m^2)$ dB. However, the resulting waveform has a spectral line at the carrier, which can be found using a conventional phase-locked loop.

Comparison of digital modulation schemes

The simple bandpass signalling schemes discussed so far have relative strengths and weaknesses. These are summarised here.

1 Amplitude-shift keying

The net probability of error for a coherently detected OOK system is

$$P_\epsilon = \text{erfc}\sqrt{\frac{E}{2\eta}},$$

where E is the bit energy on transmission of a mark. This expression is relevant if the peak power is the important design parameter. If marks and spaces occur with equal probability, then this can be written as

$$P_\epsilon = \text{erfc}\sqrt{\frac{E_b}{\eta}},$$

with E_b the average bit energy. Because of the presence of a large carrier component, noncoherent detection of OOK is also possible, so simple envelope detection can be used in the receiver. Synchronous detection offers only about a 1dB improvement over envelope detection.

The PSD of ASK is centered at ω_c, and has an identical shape to the corresponding on-off keyed baseband signal. Since the bandwidth has been doubled by the modulation, the theoretical maximum bandwidth efficiency is 1bps/Hz.

Transmitters for ASK are easy to build, as are noncoherent receivers. OOK systems are often used in short-range minature telemetry. The decision threshold in the receiver does however have to be adjusted with changes in received signal levels, usually by means of an automatic gain control circuit.

2 Frequency-shift keying

The probability of error for coherent FSK is

$$P_\epsilon = \text{erfc}\sqrt{\frac{E}{\eta}}.$$

Since the signal is active all the time, this can be expressed in terms of the average bit energy E_b as

$$P_\epsilon = \text{erfc}\sqrt{\frac{E_b}{\eta}}.$$

In terms of average power required, the performance of FSK is therefore the same as for ASK. However, in terms of *peak* power, FSK has a 3dB advantage over ASK.

FSK systems operate symmetrically about a zero decision-threshold regardless of the carrier signal strength, so threshold adjustments need not be made. Additionally, there is little difference in complexity in FSK transmitters over ASK. Receiver complexity may vary, however, according to whether coherent or noncoherent detection is used.

Noncoherent detection of FSK is quite simple to perform, and is popular for low-to-medium data transmission rates. However, the frequencies used must then satisfy the condition $2\Delta f T \gg 1$, so that the peaks in the PSD are well-separated. The bandwidth required in this case is $2\Delta f + 2B$, where B is the baseband bandwidth.

For coherent detection Δf can be made as small as desired, but cases for $2\Delta f T < \frac{1}{2}$ result in a S/N penalty. Bandwidths for FSK transmission intended for coherent demodulation are typically equal to or slightly greater than those used for ASK.

3 Phase-shift keying

The error probability for PSK is

$$P_\epsilon = \text{erfc}\sqrt{\frac{2E}{\eta}},$$

or

$$P_\epsilon = \text{erfc}\sqrt{\frac{2E_b}{\eta}}.$$

Thus PSK systems require less transmitted power for a given probability of error than ASK or FSK systems.

Synchronous detection of PSK signals is required, due to the absense of a large carrier component. Carrier recovery is therefore more complex and expensive. DPSK systems are often a good compromise, offering simpler circuitry at a small performance cost. The PSD of a PSK waveform is centered around ω_c, and has an identical shape to that of the double sideband modulating spectral density.

For PSK with $\Delta\theta < \pi/2$, there is a carrier component and the PSD has an impulse at the carrier frequency. The carrier component need not be large with respect to the sidebands. The theoretical bandwidth efficiency of PSK systems is 1bps/Hz.

4 Error performance curves

These can be found in Stremler. The order of performance, from worst to best, is

- Noncoherent ASK
- Coherent ASK

- Noncoherent FSK
- Coherent FSK
- DPSK
- Coherent PSK.

The three most widely used digital modulation methods for communication systems are PSK, DPSK, and noncoherent FSK.