Frequency-shift keying (FSK)
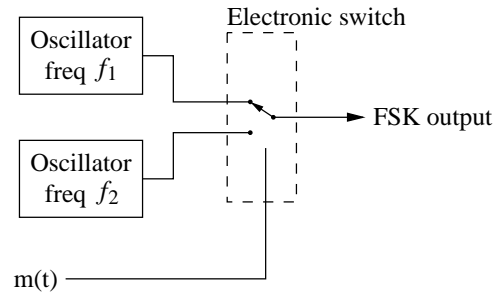
In frequency-shift keying, the signals transmitted for marks (binary ones) and spaces (binary zeros) are

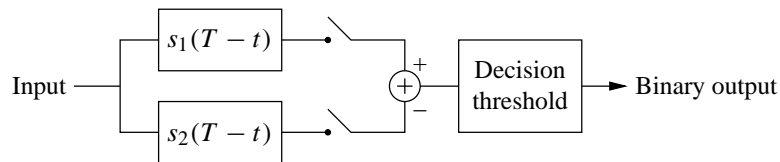$$s_1(t) = A\cos(\omega_1 t + \theta_c), \qquad 0 < t \le T$$
$$s_2(t) = A\cos(\omega_2 t + \theta_c), \qquad 0 < t \le T$$

respectively. This is called a **discontinuous phase** FSK system, because the phase of the signal is discontinuous at the switching times.

A signal of this form can be generated by the following system:

If the bit intervals and the phases of the signals can be determined (usually by the use of a phase-lock loop), then the signal can be decoded by two separate matched filters:

The first filter is matched to the signal $s_1(t)$, and the second to $s_2(t)$. Under the assumption that the signals are mutually orthogonal, the output of one of the matched filters will be E and the other zero (where E is the energy of the
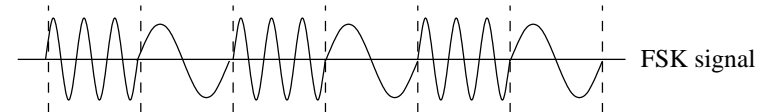
signal). Decoding of the bandpass signal can therefore be achieved by subtracting the outputs of the two filters, and comparing the result to a threshold.

If the signal $s_1(t)$ is present then the resulting output will be $+E$, and if $s_2(t)$ is present it will be $-E$. Since the noise variance at each filter output is $E\eta/2$, the noise in the difference signal will be doubled, namely $\sigma^2 = E\eta$. Since the overall output variation is $2E$, the probability of error is

$$P_\epsilon = \text{erfc}\left(\frac{2E}{2\sqrt{E\eta}}\right) = \text{erfc}\sqrt{\frac{E}{\eta}}.$$

The overall performance of a matched filter receiver in this case is therefore the same as for ASK.

The frequency spectrum of an FSK signal is difficult to obtain — this is a general characteristic of FM signals. However, some rules of thumb can be developed. Consider the case where the binary message consists of an alternating sequence of zeros and ones. If the two frequencies are each multiples of $1/T$ (eg. $f_1 = m/T$ and $f_2 = n/T$) and are synchronised in phase, then the FSK wave is a periodic function:
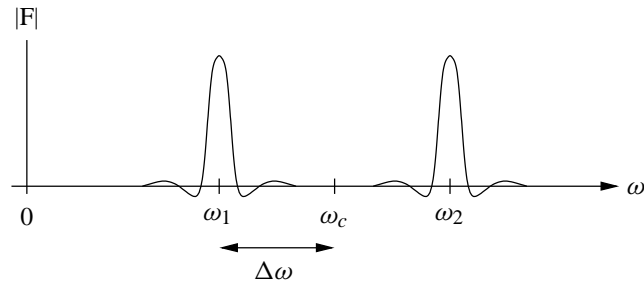

This can be viewed as the linear superposition of two OOK signals, one delayed by T seconds with respect to the other. Since the spectrum of an OOK signal is

$$F_c(\omega) = \int_{infty}^{\infty} Am(t)\cos(\omega_c t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} A/2m(t)[e^{j\omega_c t} + e^{-j\omega_c t}]dt$$
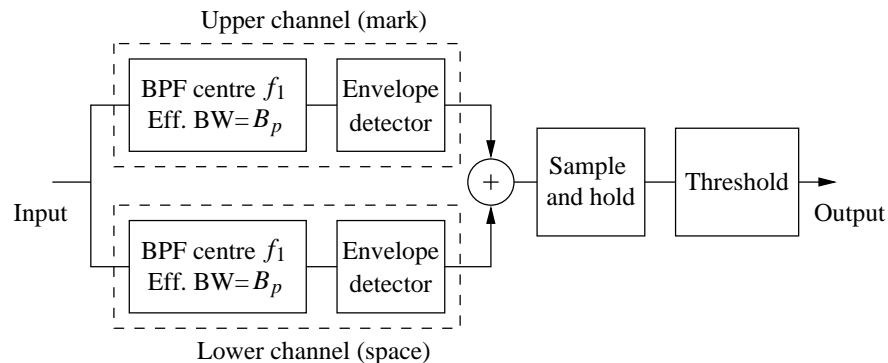$$= A/2[M(\omega - \omega_c) + M(\omega + \omega_c)],$$

where $M(\omega)$ is the transform of the baseband signal $m(t)$, the spectrum of the FSK signal is the superposition of two of these spectra, one for $\omega_1 = \omega_c - \Delta\omega$

and the other for $\omega_2 = \omega_c + \Delta\omega$. An example is shown below for positive frequencies under the assumption that $2\Delta f T \gg 1$:

The bandwidth of the periodic FSK signal is then $2\Delta f + 2B$, with B the bandwidth of the baseband signal.

Nonsynchronous or envelope detection can be performed for FSK signals. In this case the receiver takes the following form:

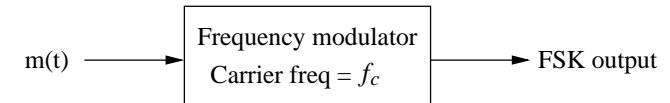The bit error probability can be shown to be

$$p_\epsilon = \frac{1}{2}e^{-E/2\eta},$$

which under normal operating conditions corresponds to less than a 1dB penalty over coherent detection. In practice almost all FSK receivers are of this

form.

In order for envelope detection to be successful, the peaks in the frequency domain at $\omega_c - \Delta\omega$ and $\omega_c + \Delta\omega$ must be widely separated with respect to the bandwidth of the baseband signal. This requires $2\Delta f T > 1$.

A more practical alternative to discontinuous-phase FSK systems are **continuous-phase** FSK systems, where a polar binary baseband signal is provided as the input to a voltage-controlled oscillator (VCO):

In this case the FSK signal can be represented by

$$s(t) = A\cos\left[\omega_c t + D_f \int_{-\infty}^{t} m(\lambda)d\lambda\right].$$

Overly sharp transitions in the phase of the output signal can be restricted by band-limiting the input to the VCO.

Note that FSK is not true frequency modulation, and does not provide the wide-band noise reduction properties associated with FM.

Example

A rectangular-pulse polar baseband signal is used to modulate an RF carrier in FSK. If the baseband signal has a data rate of 200 kbit/sec and the two RF frequencies are 150 kHz apart, determine the bandwidth.

The bit period in the baseband signal is $T = 1/200000$ seconds, and the baseband pulses are rectangular. The bandwidth of the baseband signal (to the first null) is given by $B = 1/(2T)$. For the RF components, $2\Delta f = 150$ kHz. The bandwidth of the FSK signal is therefore

$$2\Delta f + 2B = 150\,\text{kHz} + 200\,\text{kHz} = 350\,\text{kHz}.$$