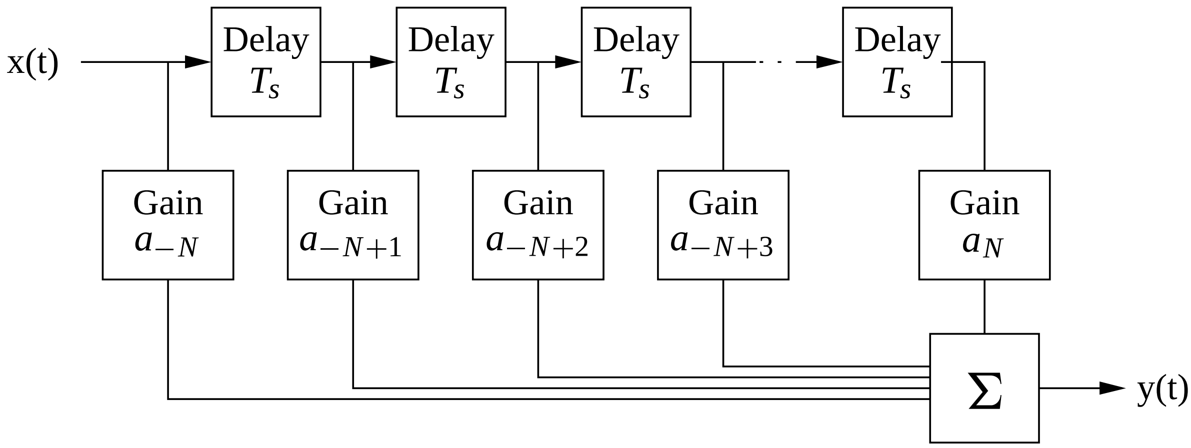
Equalisation

Practically, the frequency response of a channel is not known sufficiently accurately to allow for the design of a receiver with zero ISI for all time. The output from the channel is therefore usually filtered in such a way that the channel-induced distortion is corrected. This process is called **equalisation**, and is often implemented using a **transversal filter** structure:



As shown, the transversal filter consists of a tapped delay line where the tap outputs are multiplied by gain factors. The resulting products are summed to produce the filter output. The transversal structure is popular for this application because it is easy to analyse and design, and has efficient means of implementation. The input $x(t)$ is from the channel, and the equaliser is designed to provide an output waveform $y(t)$ that has zero ISI.

The tap coefficients a_k are set to subtract the effects of interference from symbols that are adjacent in time to the current symbol. Consider the case of $2N + 1$ taps with coefficients $a_{-N}, \ldots, a_0, \ldots, a_N$. Letting $x_k = x(kT_s)$ and $y_k = y(kT_s)$, the equaliser output at the sample points is

$$y_k = \sum_{n=-N}^{N} a_n x_{n-k}.$$

Note that the filter as written is not causal, but can be made so by delaying the output by N samples, and writing y_{k+N} for y_k.

The criterion for selecting the filter coefficients is typically based on minimising either the peak or mean-square distortion. Minimising peak distortion can be achieved choosing a_k so that the equaliser output is forced to zero at N sample points on either side of the desired pulse. Under this condition, we require that the equaliser output satisfy

$$y_k = \begin{cases} 1 & k = 0 \\ 0 & k = \pm 1, \dots, \pm N. \end{cases}$$

This formulation leads to a **zero-forcing equaliser** which is guaranteed to produce zero ISI for N samples before and N samples after the peak of the pulse.

The conditions on the zero-forcing equaliser can be written as

$$\sum_{n=-N}^{N} a_n x_{k-n} = \begin{cases} 1, & k = 0 \\ 0, & k = \pm 1, \dots, \pm N. \end{cases}$$

This represents $2N + 1$ equations in the $2N + 1$ unknowns $a_{-N}, \dots, a_0, \dots, a_N$. The system of equations can be represented in matrix form as

$$\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} x_0 & x_{-1} & x_{-2} & \cdots & x_{-2N} \\ x_1 & x_0 & x_{-1} & \cdots & x_{-2N+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{2N} & x_{2N-1} & x_{2N-2} & \cdots & x_0 \end{pmatrix} \begin{pmatrix} a_{-N} \\ a_{-N+1} \\ \vdots \\ a_0 \\ \vdots \\ a_N \end{pmatrix}.$$

Thus the equations take the form $\mathbf{H_e} = \mathbf{X_c a}$, where $\mathbf{H_e}$ and $\mathbf{a}$ are the column vectors on the left and right hand sides respectively, and $\mathbf{X_c}$ is the matrix as indicated. The solution

$$\mathbf{a} = \mathbf{X_c}^{-1}\mathbf{H_e}$$

therefore provides the set of values for the tap coefficients of the transversal filter that gives zero ISI.

The N-tap equaliser can only force the ISI to zero for N samples before and after the current time. In general it can therefore not completely eliminate ISI. However, the ISI beyond N samples is usually negligible since the time response has decayed to an insignificant amplitude over that range.

There are two general types of automatic equalisation. In **preset equalisation** the transmitter emits a training sequence that is compared by the receiver to a locally generated sequence. The differences between the two are used to set the filter coefficients. The initial training session must be repeated periodically, usually after any break in transmission. Also, if the channel is time-varying then performance suffers.

The taps can be adjusted to minimise the mean-square error (MSE) between the received signal y_k and the known training signal c_k. For K samples the MSE is

$$\overline{e^2} = \frac{1}{K}\sum_{k=1}^{K}(y_k - c_k)^2.$$

At the optimal tap weight setting the error is minimised, so

$$\frac{d\overline{e^2}}{da_n} = 0$$

for all the tap weights a_n. Differentiating gives the necessary condition

$$\frac{2}{K}\sum_{k=1}^{K}(y_k - c_k)x_{k-n} = 0, \qquad n = 0, \pm 1, \ldots, \pm N.$$

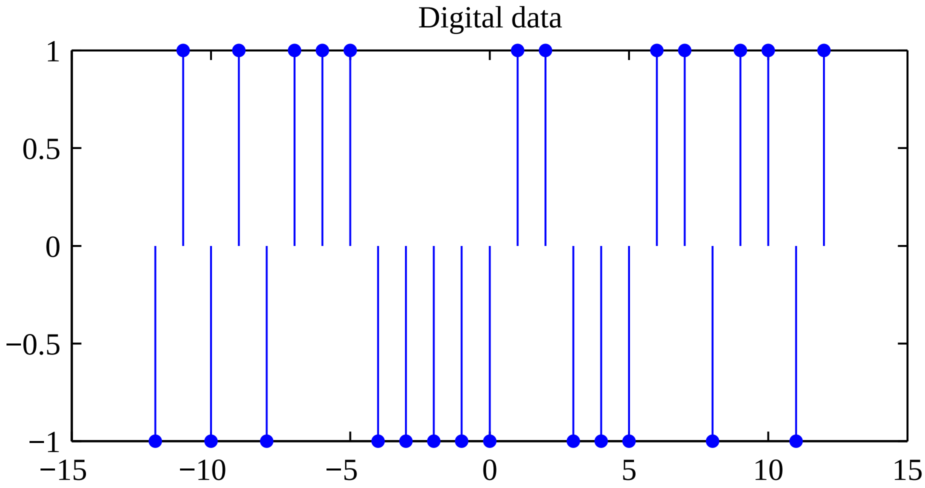
Note that this is equivalent to the condition that

$$R_{ex}(n) = 0, \qquad n = 0, \pm 1, \ldots, \pm N,$$

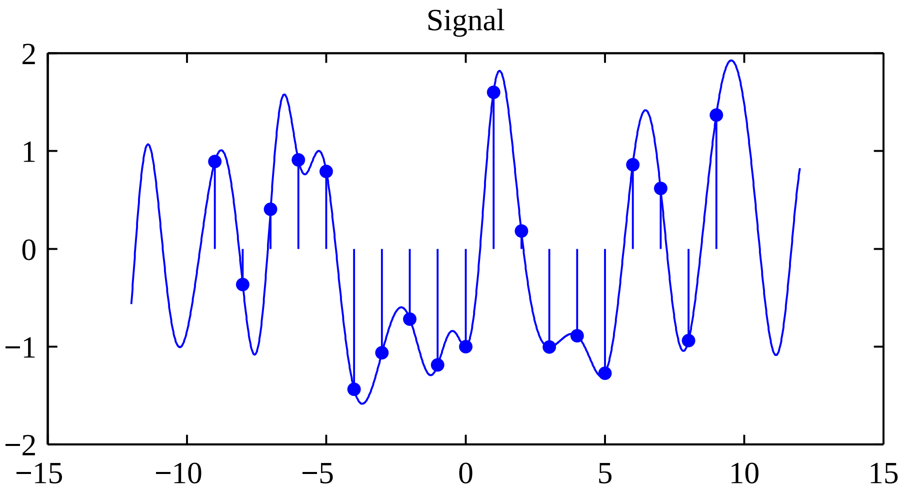
where $R_{ex}(n)$ is the cross correlation between the error sequence and the input sequence. We therefore require that these sequences be uncorrelated. Weight adjustment procedures can be developed to enforce this condition.

In **adaptive equalisation**, the coefficients are continually and automatically adjusted from the transmitted data. This performs well if the channel error performance is satisfactory, but in poor performance environments the received channel errors may not allow the coefficient adjustment algorithm to converge. A common solution is to use preset equalisation initially, and then to switch to adaptive equalisation once normal transmission begins.

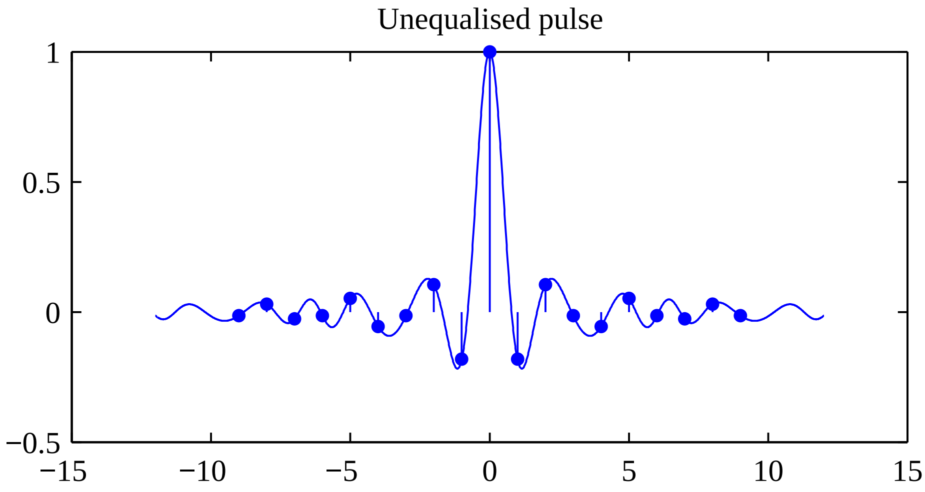
Example:

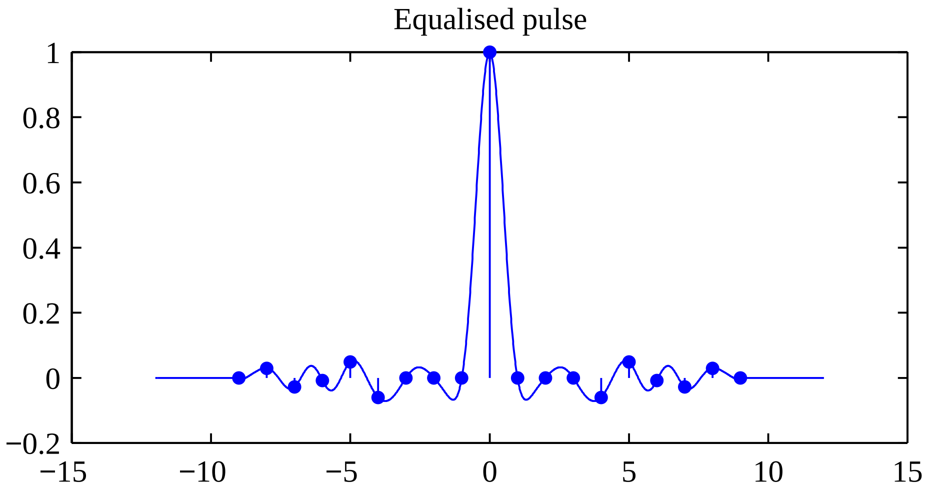


Digital data
1
0.5
0
−0.5
−1
−15
−10
−5
0
5
10
15



Signal
2
1
0
−1
−2
−15
−10
−5
0
5
10
15


Unequalised pulse
1
0.5
0
−0.5
−15
−10
−5
0
5
10
15


Equalised pulse
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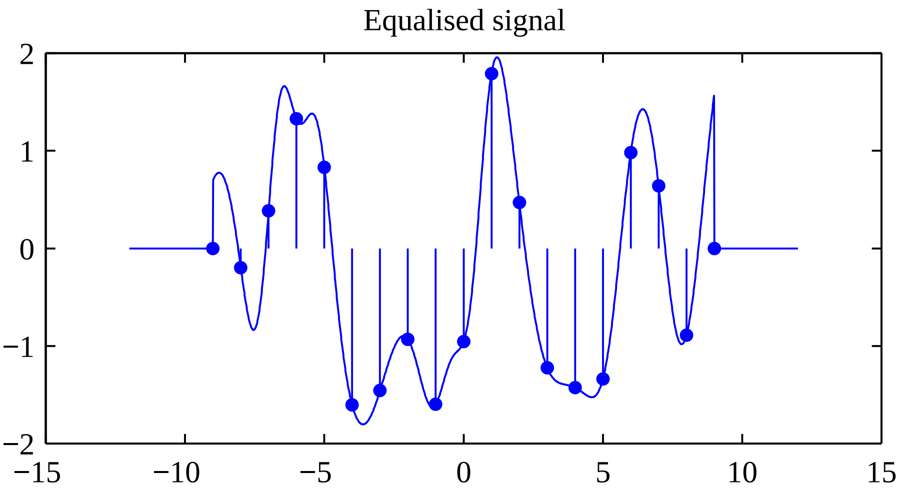
0.8
0.6
0.4
0.2
0
−0.2
−15
−10
−5
0
5
10
15


Equalised signal
2
1
0
−1
−2
−15
−10
−5
0
5
10
15