EEE401F EXAM
DIGITAL SIGNAL PROCESSING

University of Cape Town
Department of Electrical Engineering

June 2003
3 hours

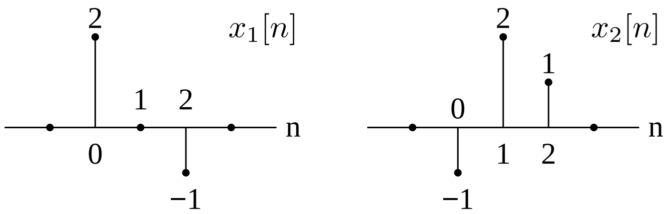
Information

- The exam is closed-book.
- There are two parts to this exam.
- **Part A** has *seven* questions totalling 70 marks. You must answer all of them.
- **Part B** has *three* questions. You must choose *two* of them, to make up a further 30 marks.
- You have 3 hours.
- A formula sheet for the radar section is attached to the end of this paper.

PART A

Answer all of the following questions.

1. You are given the two signals $x_1[n]$ and $x_2[n]$ shown below. The signal values are zero outside of the range of n shown:



   (a) Use graphical methods to find the linear convolution of $x_1[n]$ and $x_2[n]$.

   (b) Use the z-transform convolution property

$$x_1[n] * x_2[n] \overset{\mathcal{Z}}{\longleftrightarrow} X_1(z)X_2(z)$$

   to find the linear convolution of the two signals.

(10 marks)

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2. Consider the constant coefficient difference equation

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n-1],$$

   where $x[n]$ is the input and $y[n]$ the output to the system.

   (a) Find an expression for the system function $H(z)$ corresponding to the difference equation.

   (b) What are the choices for the region of convergence of the system function? Which ROCs correspond to a stable and a causal system respectively?

   (c) Using a partial fraction expansion and the z-transform pair

$$a^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| > |a|,$$

   find an expression for the causal impulse response of the system.

(10 marks)

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3. Consider a filter with the following system function:

$$H(z) = \frac{z^2 + \frac{1}{4}z}{\left(z - \frac{1}{2}\right)^2}, \qquad |z| > \frac{1}{2}.$$

(a) Using the z-transform properties

$$na^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{az^{-1}}{(1 - az^{-1})^2}, \qquad |z| > |a|$$

and

$$x[n - n_0] \overset{\mathcal{Z}}{\longleftrightarrow} z^{-n_0} X(z),$$

find the impulse response of the system.

(b) Find a difference equation relating the input $x[n]$ to the output $y[n]$ of the filter.

(10 marks)

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4. (a) Use the power series method to find the first 5 values of the causal discrete-time sequence corresponding to the z-transform

$$X(z) = \frac{z - 1}{\left(z - \frac{1}{2}\right)^2}.$$

(b) What is the magnitude and phase of the discrete-time Fourier transform of this signal at $\omega = \pi/2$?

(10 marks)

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5. Consider the system function

$$X(z) = \frac{z}{z + 0.8},$$

with a region of convergence $|z| > 0.8$.

(a) Draw a pole-zero plot of the system, and indicate the region of convergence.

(b) Use geometric techniques to make a rough sketch of the magnitude and phase of the transfer function of the system.

(c) What type of filter does the system represent?

(d) Is the system causal? Is the system stable?

(10 marks)

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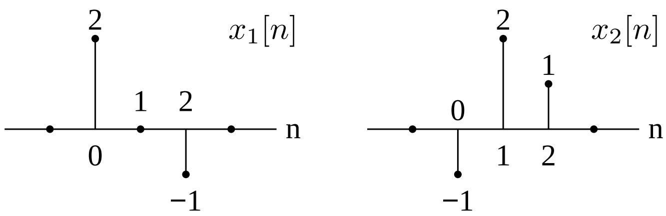
6. Consider the two signals $x_1[n]$ and $x_2[n]$ shown below. The signal values are zero outside of the range of n shown:



(a) Using either graphical or direct algebraic methods to find the 4-point circular convolution of $x_1[n]$ and $x_2[n]$.

(b) Use the discrete Fourier transform (DFT) to find the 4-point circular convolution by finding the DFT of each of the signals, multiplying the result, and taking the inverse DFT.

(10 marks)

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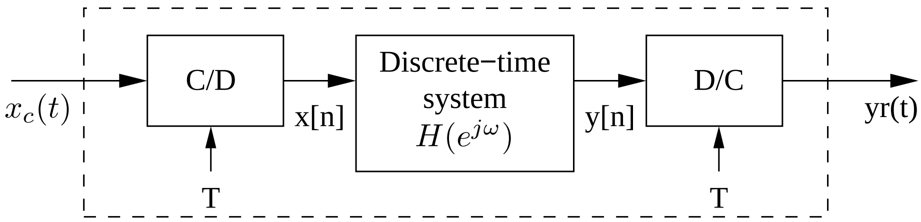
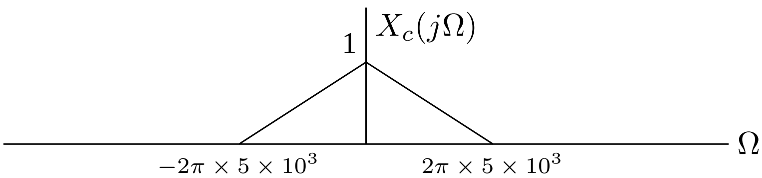
7. An analogue filter is implemented using discrete-time processing as follows:



The continuous-time input signal $x_c(t)$ has Fourier transform



(a) Sketch the discrete-time Fourier transform of $x[n]$ if the sampling frequency is $1/T = 1.5 \times 10^4$ Hz.

(b) What is the largest value that T that avoids aliasing in the system? Sketch the DTFT of $x[n]$ for this case.

(c) What are two advantages of using a digital system to do analogue filtering?

(d) What are two disadvantages of using a digital system to do analogue filtering?

(10 marks)

PART B

Answer *two* of the following three questions.

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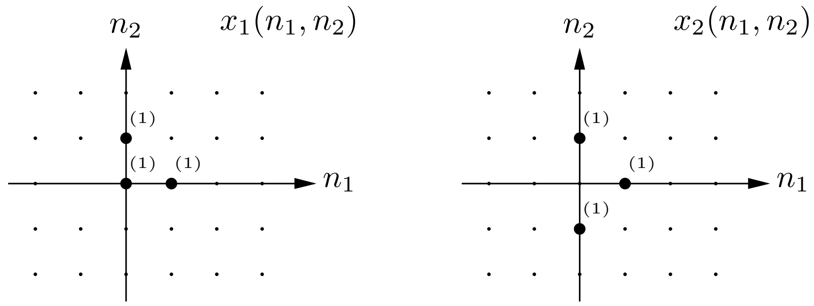
1. **Multidimensional signal and image processing**

   (a) Use the method of your choice to find the 2-D convolution between the following two signals:



(5 marks)

   (b) Explain the role of a point-spread function (PSF) in the image formation process, and describe how 2-D convolution plays a role. How does the PSF change as the imaging system is defocused? Finally, what is the MTF, and how does it change with defocus?

(5 marks)

   (c) Describe how you would go about detecting vertical edges (like the edges of buildings) in an image. Describe the role of 2-D convolution (or filtering) in the process, and discuss the choice of filter parameters. How would you reduce the effects of noise in the edge detection process?

(5 marks)

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2. **Speech processing**

(a) What information is contained in a speech signal? List at least 3 kinds of information.

(2 marks)

(b) What are some of the techniques or methods that are used to extract the information from the speech signal?

(2 marks)

(c) A spectrogram can be used to 'visualize' the speech signal.

i. How is the spectrogram constructed? (1 mark)

ii. How many dimensions are shown in the spectrogram? (1 mark)

iii. What are the dimensions of the spectrogram? (1 mark)

(d) Linear Predictive Coding (LPC) coefficients are used as the basis of GSM (cellular phone) speech codecs, What are the three major weaknesses of the LPC vocoders?

(2 marks)

(e) Derive expressions for the optimal LPC coefficients α_i in the model

$$\tilde{s}[n] = \sum_{i=1}^{p} \alpha_i s[n-i],$$

given a sample of values $s[1], \ldots, s[n]$. Assume that the optimisation criterion is the minimisation of the mean-squared prediction error.

(6 marks)

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3. **Radar signal processing**

(a) Draw a neatly labelled block diagram of a coherent radar system showing (i) transmitter chain, (ii) receiver with an I-Q down converter (iii) appropriate sampling into a digital signal processor.

(2 marks)

(b) Draw a neatly labelled block diagram of the equivalent analytic signal model of the radar.

(2 marks)

(c) Illustrate with the aid of sketches of the frequency spectra, how the signals in the system are related, particularly

i. the impulse response of the scene $\xi(t) \leftrightarrow \xi(f)$

ii. the baseband form $p(t) \leftrightarrow P(f)$ and the transmitted rf pulse $v_{tx}(t) \leftrightarrow V_{tx}(f)$

iii. the complex baseband signal $v_{bb}(t) \leftrightarrow V_{bb}(f)$.

(3 marks)

(d) A digital signal processing algorithm must be developed for pulse compression and display of the received echoes. The transmitted pulse is a chirp pulse with bandwidth of 150 MHz. What digital signal processing steps would you apply to obtain a focussed image of the scene, considering that you would like to optimise signal to noise ratio and keep the sidelobes low? The processor operates on the complex baseband received signal. Sketch the point target response at the output of your processor.

(4 marks)

(e) What properties of the transmitted pulse determine (i) the resolution of the radar (ii) the SNR of the processed range data.

(2 marks)

(f) What are the primary advantages of transmitting a CHIRP pulse as opposed to a monochrome pulse?

(2 marks)
