EEE4001F: Digital Signal Processing

Class Test 2

22 April 2010

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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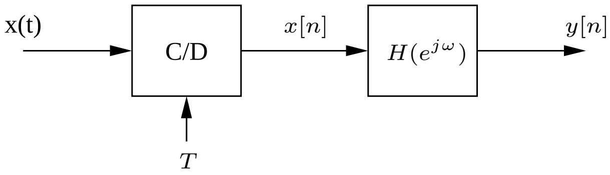
1. (5 marks) Consider the system below



where $T = 0.001$s and

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq 0.5\pi \\ 0 & \text{otherwise} \end{cases}$$

for $-\pi \leq \omega \leq \pi$. Find the output $y[n]$ if the input is $x(t) = \cos(400\pi t) + \cos(600\pi t)$.

2. (5 marks) Consider the following discrete-time signals $x[n]$ and $y[n]$:

$$x[n] = 0.2\cos(0.2\pi n) \quad \text{and} \quad y[n] = 0.2\sin(0.2\pi n).$$

(a) Show that the 10-point DFT of $x[n]$ is $X[k] = \delta[k-1] + \delta[k-9]$ over the range $k = 0, \ldots, 9$.

(b) Assuming that the 10-point DFT of $y[n]$ is $Y[k] = -j(\delta[k-1] - \delta[k-9])$, use the DFT to determine a *closed-form* expression for the 10-point circular convolution of $x[n]$ and $y[n]$.

3. (5 marks) A stable system is characterised by the following LCCDE:

$$y[n+2] - y[n+1] + \frac{1}{2}y[n] = x[n+1].$$

(a) Draw a pole-zero plot of the system.

(b) Roughly sketch the magnitude response of the system.

(c) Assuming the system response represents a band-pass filter at a frequency of $\pi/4$ radians/sample, what is the centre frequency of the passband if an analog signal is sampled at 12kHz before filtering?

4. (5 marks) A particular DSP system is sampled at 48kHz, and requires a highpass filter with a passband ripple of 0.1dB, a transition band of 200Hz, stopband attenuation of 60dB, and a cutoff frequency of 1200Hz. Sketch the appropriate design constraints that the filter must satisfy, specifying parameter values where appropriate. Your frequency axis should be in units of radians per sample.

Fourier transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(e^{j\omega})$, $Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \|\omega\| < \omega_c \\ 0 & \omega_c < \|\omega\| \leq \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$\begin{cases} a^n & 0 \leq n \leq N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2 z^{-2}}$	$\|z\| > r$