

EEE4001F: Digital Signal Processing

Class Test 2

30 April 2008

Name:

Student number:

Information

- The test is closed-book.
 - This test has *four* questions, totalling 20 marks.
 - Answer *all* the questions.
 - You have 45 minutes.
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1. (5 marks) Find $w[n] = x[n] * y[n]$ with

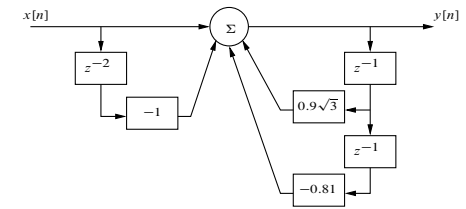
$$x[n] = (-1)^n \quad \text{and} \quad y[n] = \frac{\sin(\pi n/3)}{\pi n}.$$

2. (5 marks) Consider the following discrete-time signal:

$$x[n] = \begin{cases} \sin\left(\frac{n\pi}{4}\right) & \text{when } n/2 \text{ is an integer} \\ 0 & \text{otherwise.} \end{cases}$$

Calculate and sketch the 8-point DFT (magnitude and phase) of the first 8 samples of $x[n]$, i.e. $x[0], x[1], \dots, x[7]$. Show and motivate your calculations.

3. (5 marks) Consider the following LTI system:



(a) Show that the system function is

$$H(z) = \frac{(z-1)(z+1)}{(z-0.9e^{j\pi/6})(z-0.9e^{-j\pi/6})}.$$

(b) Sketch the magnitude frequency response $|H(e^{j\omega})|$ over the range $0 \leq \omega \leq 2\pi$. Indicate calculated amplitudes at $\omega = 0$, $\omega = \frac{\pi}{6}$, and $\omega = \pi$.

4. (5 marks) One of the simplest filters is a backward-difference system, where

$$y[n] = x[n] - x[n - 1].$$

Using sketches, notes, and equations, justify why this is a highpass filter.

Fourier transform properties

Sequences $x[n], y[n]$	Transforms $X(e^{j\omega}), Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
1 ($-\infty < n < \infty$)	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\omega + 2\pi k)$
$a^n u[n]$ ($ a < 1$)	$\frac{1}{1 - ae^{-j\omega}}$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\omega + 2\pi k)$
$(n + 1)a^n u[n]$ ($ a < 1$)	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \omega < \omega_c \\ 0 & \omega_c < \omega \leq \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z < 1$
$\delta[n - m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z > a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z < a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z > a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z < a $
$\begin{cases} a^n & 0 \leq n \leq N - 1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$ z > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1 - r \cos(\omega_0)z^{-1}}{1 - 2r \cos(\omega_0)z^{-1} + r^2 z^{-2}}$	$ z > r$