

# EEE4001F: Digital Signal Processing

## Class Test 1

20 March 2008

Name:

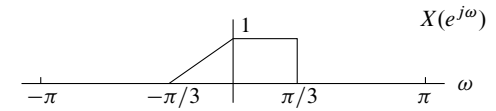
Student number:

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### Information

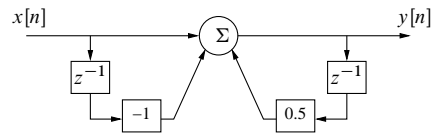
- The test is closed-book.
  - This test has *four* questions, totalling 20 marks.
  - Answer *all* the questions.
  - You have 45 minutes.
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1. (5 marks) A sequence  $x[n]$  has a zero-phase DTFT  $X(e^{j\omega})$  given below:



Sketch the DTFT of the sequence  $y[n] = x[-n]e^{-j\pi n/3}$ .

2. (5 marks) Consider the following LTI system:



Determine a closed-form expression for the response  $y[n]$  of this system to the following input signal:

$$x[n] = \begin{cases} 1 & n \geq 4 \\ 0 & \text{otherwise} \end{cases}$$

if the system is causal and initially at rest.

3. (5 marks) Consider the following discrete-time signal  $x[n]$ :

$$x[n] = \begin{cases} n + 1 & 0 \leq n \leq 3 \\ 4 & n \geq 4 \\ 0 & \text{otherwise.} \end{cases}$$

- Determine the z-transform  $X(z)$  of  $x[n]$ , and represent it as a ratio of polynomials in  $z^{-1}$ .
- What is the region of convergence (ROC) of this z-transform?

4. (5 marks) Consider the continuous-time signal

$$x(t) = \sin(400\pi t + \pi).$$

The discrete-time signal  $x[n]$  is obtained by sampling  $x(t)$  at  $t = n/f_s$  with a sampling frequency  $f_s = 1000$  Hz. Which one of the following continuous-time signals will yield the same sample values when sampled at the same sampling instants? Show and motivate your calculations.

- (a)  $\sin(600\pi t)$
- (b)  $-\sin(1000\pi t)$
- (c)  $\sin(1400\pi t)$
- (d)  $\sin(1600\pi t)$

### Fourier transform properties

Sequences $x[n], y[n]$	Transforms $X(e^{j\omega}), Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$	Modulation

### Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
1 $(-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n]$ $( a  < 1)$	$\frac{1}{1 - ae^{-j\omega}}$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n + 1)a^n u[n]$ $( a  < 1)$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 &  \omega  < \omega_c \\ 0 & \omega_c <  \omega  \leq \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

### Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All $z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
$\delta[n - m]$	$z^{-m}$	All $z$ except 0 or $\infty$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $
$\begin{cases} a^n & 0 \leq n \leq N - 1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$ z  > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z  > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1 - r \cos(\omega_0)z^{-1}}{1 - 2r \cos(\omega_0)z^{-1} + r^2 z^{-2}}$	$ z  > r$