EEE4001F: Digital Signal Processing

Class Test 2

26 April 2007

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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1. (5 marks) A stable LTI system is characterised by the following z-transform:

$$H(z) = \frac{1 + z^{-2}}{1 + 0.81z^{-2}}$$

   (a) Sketch the magnitude of the frequency response

   (b) Calculate the magnitude and the phase of the frequency response at $1/2$ the sampling frequency.

Multiply $H(z)$ by z^2 and factorise:

$$H(z) = \frac{(z + j)(z - j)}{(z + 0.9j)(z - 0.9j)}.$$

Plot poles and zeros, and use graphical methods to obtain magnitude response. Sampling frequency is $\omega = 2\pi$, so $1/2$ sampling frequency is Nyquist rate $\omega = \pi$. Either substitute $z = e^{j\pi}$ into above and calculate response magnitude and phase, or use graphical methods again, with exact lengths and angles of zero vectors and pole vectors.

2. (5 marks) The following signals are defined on the interval $n = 0, \ldots, 7$:

$$x_1[n] = (1/2)^n \qquad x_2[n] = (-1)^n.$$

(a) Find a closed-form expression for the 8-point DFT of $x_1[n]$.

(b) Find the 8-point DFT of $x_2[n]$.

(c) Using above results, compute the 8-point circular convolution $y[n] = x_1[n] \circledS x_2[n]$.

(a) The DFT is

$$X_1[k] = \sum_{k=0}^{7} (1/2)^n e^{-j\frac{2\pi}{8}kn} = \sum_{k=0}^{7} (1/2e^{-j\frac{2\pi}{8}k})^n = \frac{1-(1/2e^{-j\frac{2\pi}{8}k})^8}{1-(1/2e^{-j\frac{2\pi}{8}k})}$$
$$= \frac{1/2}{1-(1/2e^{-j\frac{2\pi}{8}k})}$$

(b) The DFT is

$$X_2[k] = \sum_{k=0}^{7} (-1)^n e^{-j\frac{2\pi}{8}kn} = \sum_{k=0}^{7} (e^{j\pi})^n e^{-j\frac{2\pi}{8}kn} = \sum_{k=0}^{7} (e^{j\pi} e^{-j\frac{2\pi}{8}k})^n$$
$$= \frac{1-(e^{j\pi}e^{-j\frac{2\pi}{8}k})^8}{1-(e^{j\pi}e^{-j\frac{2\pi}{8}k})}.$$

The numerator is zero for all k, so $X_2[k]$ is zero except when the denominator is zero (i.e. $k = 4$). Calculating this term directly:

$$X_2[4] = \sum_{k=0}^{7} (e^{j\pi})^n e^{-j\frac{2\pi}{8}4n} = \sum_{k=0}^{7} (e^{j\pi} e^{-j\pi})^n = \sum_{k=0}^{7} (1) = 8.$$

(c) The 8-point circular convolution is the product of the DFTs: $Y[k] = X_1[k]X_2[k]$. Evidently $Y[k] = 0$ for $k \neq 4$, and $Y[4] = X_1[4]X_2[4] = (1/3)(8) = 8/3$. The required signal is the inverse DFT

$$y[n] = \sum_{k=0}^{7} Y[k] e^{j\frac{2\pi}{8}kn} = (8/3)e^{j\frac{2\pi}{8}4n} = (8/3)e^{j\pi n}.$$

3. (5 marks) Digital audio tape (DAT) drives use a sampling frequency of 48 kHz. Compact disks (CDs) use 44.1 kHz. Explain in detail how you would transfer a recording from a DAT to a CD. Give reasons and quantify any required parameters.

DAT (48 kHz) has higher sampling rate than CD, so downsampling is required.

- An initial lowpass filter is required to remove freqencies that will alias on CD. This is applied to the initial DAT data stream, and the cutoff required is $\omega_c = 44.1/48\pi$.
- We only know how to change sampling rates by integer amounts. If we first upsample by L and then downsample by M, then we require $L/M = 44.1/48$ with L, M the smallest possible integers (to save computation). Gives $L = 147$, $M = 160$.

The data rate is now appropriate for a CD.

4. (5 marks) Design a length-5 FIR bandpass filter with an antisymmetric impulse response $h[n]$ (i.e. $h[n] = -h[4-n]$ for $0 \leq n \leq 4$) satisfying the following magnitude response:

$$|H(e^{j\pi/4})| = 0.5 \qquad \text{and} \qquad |H(e^{j\pi/2})| = 1.$$

(Hint: calculate $h[n]$ and note that $h[2] = 0$.)

Antisymmetry (given) says $h[0] = -h[4]$, $h[1] = -h[3]$, $h[2] = -h[2]$, and the last expression means that $h[2] = 0$. Therefore only need to specify $h[0]$ and $h[1]$.

The Fourier transform of the 5-point impulse response is

$$\begin{aligned} H(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \\ &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} + h[4]e^{-j4\omega} \\ &= h[0] + h[1]e^{-j\omega} - h[1]e^{-j3\omega} - h[0]e^{-j4\omega} \\ &= h[0](1 - e^{-j4\omega}) + h[1](e^{-j\omega} - e^{-j3\omega}). \end{aligned}$$

Two unknowns plus two given constraints on magnitude response gives solution.