EEE401F: Digital Signal Processing

Class Test 2

25 May 2005

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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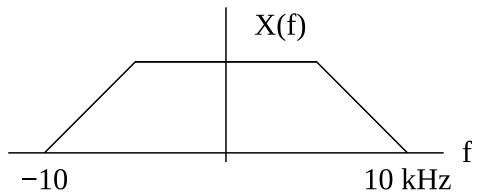
1. (5 marks) A signal has the spectrum depicted below:

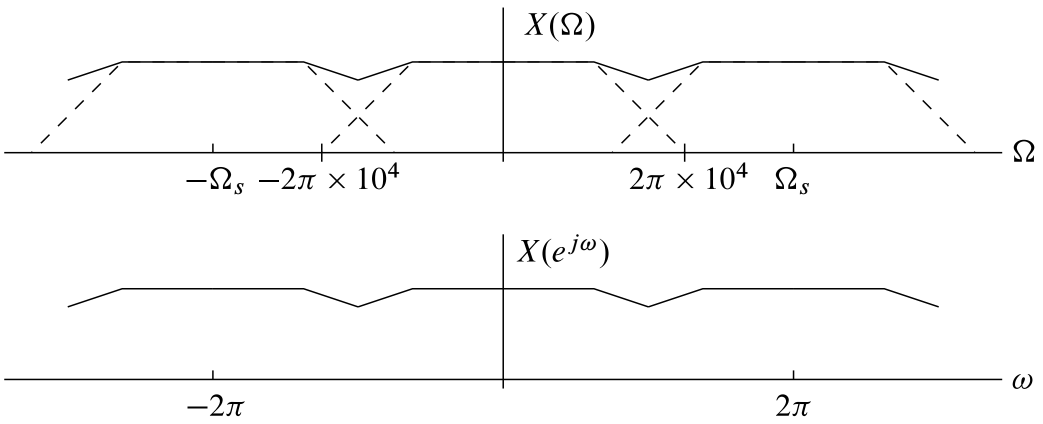


   (a) Determine the minimum sampling frequency required for perfect reconstruction.

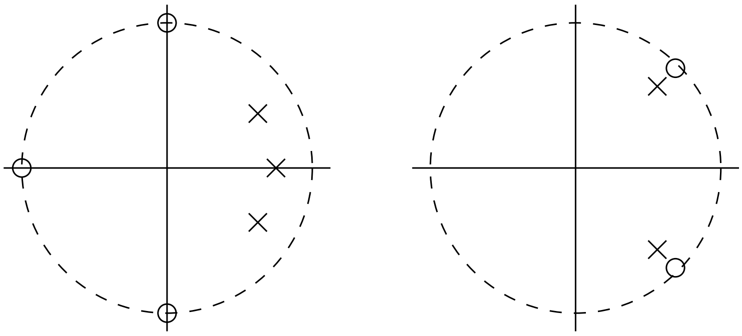
   (b) Sketch the spectrum of the sampled signal if the sampling rate is 16kHz.

For reconstruction we require samples to be taken at twice the highest frequency present in the signal, or 20kHz.

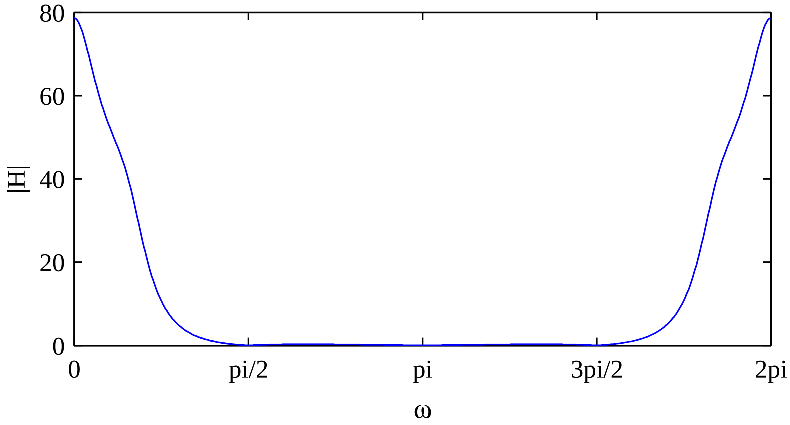
The spectrum of the discretised signal is as follows:

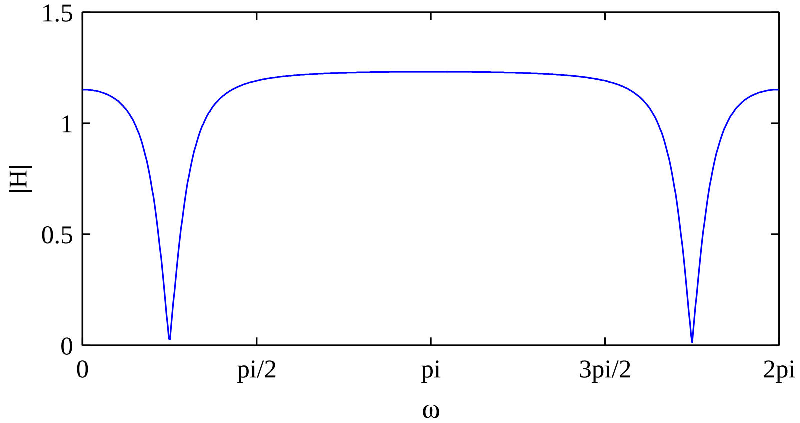


2. (5 marks) Sketch the magnitude transfer functions of the systems with the following z-plane representations:



Using graphical methods you should obtain something like the following results:





The numbers in these plots are for poles at $0.8e^{\pm j\pi/6}$ and 0.8 in the first case, and $0.8e^{\pm j\pi/4}$ in the second. However, Since no numbers were given I cannot expect numerical correctness — all that is required is the basic characteristics.

3. (5 marks) Explain, with examples and sketches, why windowing is important in spectrum estimation.

4. (5 marks) Consider the sequence $x[n] = 4\delta[n] + 3\delta[n-1] + 2\delta[n-2] + \delta[n-3]$, and let $X[k]$ be the 6-point DFT of $x[n]$.

   (a) Find find finite-length sequence $y[n]$ that has a 6-point DFT $Y[k] = W_6^{4k} X[k]$.

   (b) Find the DFT of the 6-point circular convolution of $x[n]$ with itself.

$$X[k] = \sum_{n=0}^{5} x[n] W_6^{kn} = 4W_6^{0k} + 3W_6^{1k} + 2W_6^{2k} + W_6^{3k},$$

so

$$\begin{aligned} Y[k] &= W_6^{4k} X[k] = 4W_6^{4k} + 3W_6^{5k} + 2W_6^{6k} + W_6^{7k} \\ &= 2W_6^{0k} + W_6^{1k} + 4W_6^{4k} + 3W_6^{5k}. \end{aligned}$$

and

$$y[n] = 2\delta[n] + \delta[n-1] + 4\delta[n-4] + 3\delta[n-5].$$

Six-point circular convolution in the time domain corresponds to multiplication in the 6-point DFT domain. If $w[n] = x[n] \circledN x[n]$ is the circular convolution for $N = 6$, then in the DFT domain we have

$$\begin{aligned} W[k] &= X[k]X[k] = (4W_6^{0k} + 3W_6^{1k} + 2W_6^{2k} + W_6^{3k})(4W_6^{0k} + 3W_6^{1k} + 2W_6^{2k} + W_6^{3k}) \\ &= 17W_6^{0k} + 15W_6^{1k} + 25W_6^{2k} + 20W_6^{3k} + 10W_6^{4k} + 4W_6^{5k}. \end{aligned}$$