EEE401F Class Test

11 May 2004

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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1. (5 marks) An N-sample signal $x[n]$ has the DFT $X[k]$. Write down expressions for the DFTs of the signals
   (a) $x[((n-2))_N]$
   (b) $2x[n] + x[((n+1))_N]$
   (c) $x[((-n))_N]$.

(a) A circular shift to the right by two samples corresponds to multiplication by W_n^{2k} in the DFT domain: the transform of the unshifted signal is

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} = x[0]W_N^{0k} + x[1]W_N^{1k} + \cdots + x[N-1]W_N^{(N-1)k},$$

and the shifted signal

$$\begin{aligned} X'[k] &= \sum_{n=0}^{N-1} x[((n-2))_N] W_N^{kn} \\ &= x[N-2]W_N^{0k} + x[N-1]W_N^{1k} + x[0]W_N^{2k} + x[1]W_N^{3k} + \cdots \\ &= W_N^{2k}\left(x[0]W_N^{0k} + x[1]W_N^{1k} + \cdots + x[N-2]W_N^{(N-2)k} + x[N-1]W_N^{(N-1)k}\right) \\ &= W_N^{2k} X[k]. \end{aligned}$$

(b) The DFT of the shifted signal $x[((n+1))_N]$ is $W_N^{-k}X[k]$, using an argument analogous to that used for the previous result. From the linearity of the DFT the transform of the required signal is therefore

$$X'[k] = 2X[k] + W_N^{-k}X[k] = (2 + W_N^{-k})X[k].$$

(c) Apologies — the question should have read $x^*[((-n))_N]$, and in retrospect this third part made it rather too long.

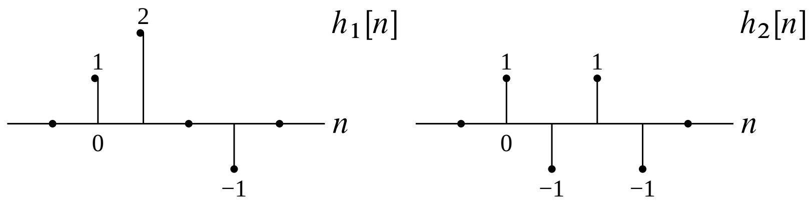
$$X'[k] = \sum_{n=0}^{N-1} x^*[((-n))_N] W_N^{kn} = x^*[0]W_N^{0k} + x^*[N-1]W_N^{1k} + \cdots + x^*[1]W_N^{(N-1)k},$$

so

$$\begin{aligned} (X'[k])^* &= x[0]W_N^{-0k} + x[N-1]W_N^{-1k} + \cdots + x[1]W_N^{-(N-1)k} \\ &= x[0]W_N^{0k} + x[1]W_N^{1k} + \cdots + x[N-1]W_N^{(N-1)k} = X[k] \end{aligned}$$

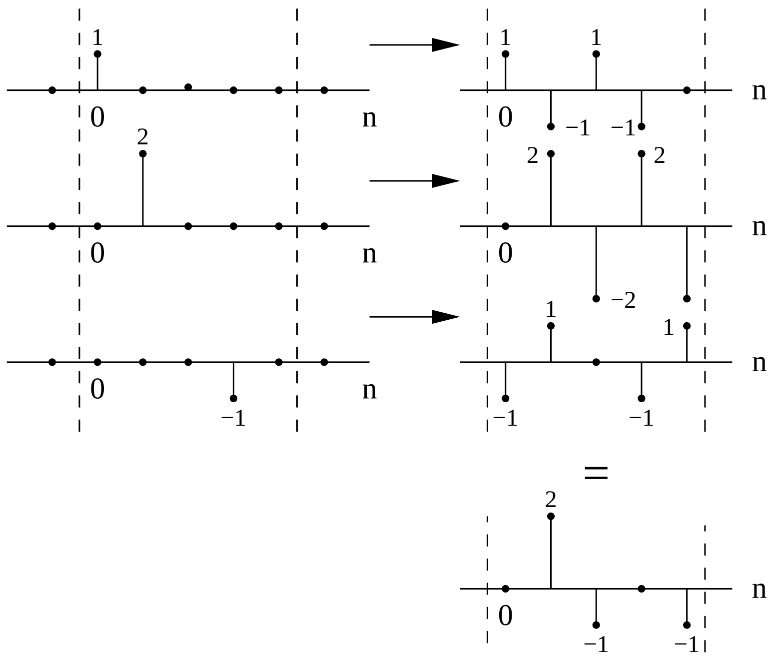
(by rearranging terms and multiplying by $W_N^{Nk} = 1$), so $X'[k] = X^*[k]$.

2. (5 marks) Using any method of your choice, find the 5-point circular convolution between the following two signals:



Explain how you would use the fast Fourier transform to obtain this result. What is the value of the output at $n = -2$?

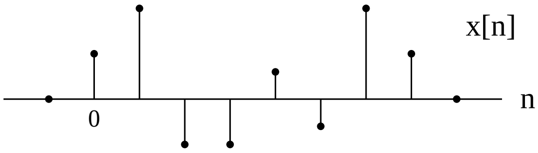
Graphically,



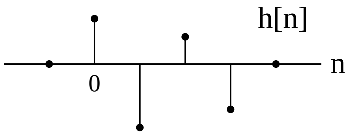
If we take the 5-point FFT (equivalently DFT) of each of the two signals, multiply them together, and invert the result, we will obtain the 5-point circular convolution.

Calling the result $y[n]$, we know that $y[-2] = \tilde{y}[-2] = y[((-2))_5] = y[3] = 0$ (circular wrap-around).

3. (5 marks) Describe in detail how you would implement fast linear convolution on an 8-point signal such as

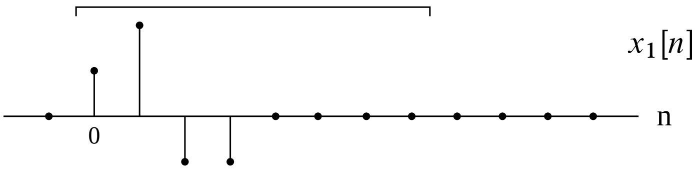


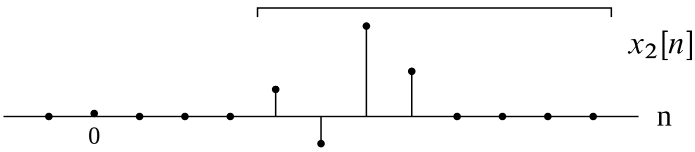
with a 4-tap FIR filter such as



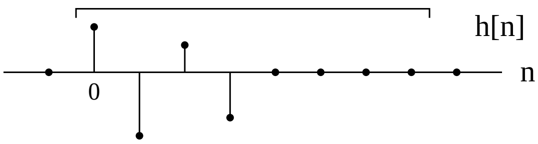
using *only* an 8-point FFT procedure.

We could use either overlap-add or overlap-save. With overlap-add, we could split the signal into two components, each with 4 nonzero elements:



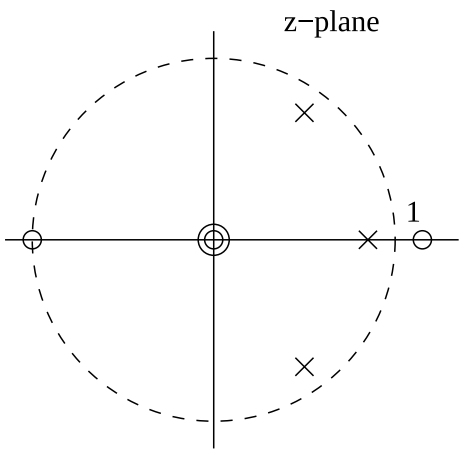


The resulting convolution will then be the sum of the convolution of each of these sequences with the filter impulse response $h[n]$:



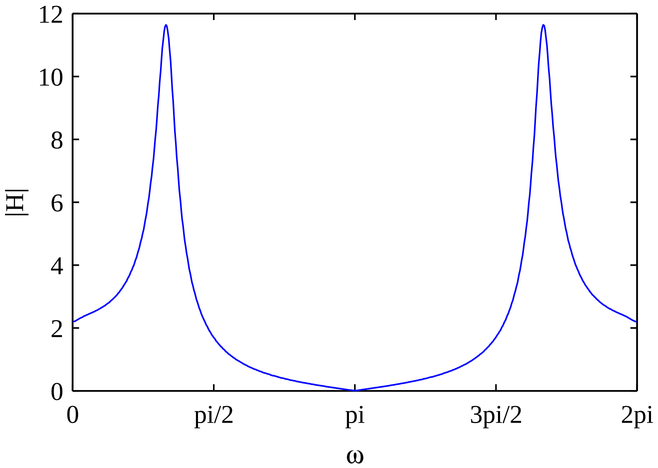
An 8-point FFT can be used to implement 8-point circular convolution of the portions of the signal indicated above, by transforming the sequences, multiplying FFT coefficients, and inverting. However, because the 8-point sequences above have sufficient trailing zeros, circular convolution is equivalent to linear convolution. We can therefore use the FFT to calculate the convolutions $x_1[n] * h[n]$ and $x_2[n+4] * h[n]$, and the required result is the sum of these sequences with the second delayed by 4 samples.

4. (5 marks) Sketch the magnitude response of the LTI system with the following pole-zero configuration:



What type of filter does this system represent? What is the approximate phase response of the system at $\omega = 0$?

Product of lengths of zero vectors divided by product of length of pole vectors should give you something like



The filter has a bandpass characteristic, with the response peaking at around $\omega = \pi/3$ (passes low frequencies, amplifies frequencies around $\pm\pi/3$, and attenuates high frequencies).

At $\omega = 0$, sum of angles of zero vectors is $0 + 0 + 0 + \pi = \pi$, and sum of angles of pole vectors is approximately $(2\pi - \frac{\pi}{3}) + 0 + \frac{\pi}{3}) = 0$. The phase is therefore π (or $-\pi$).