EEE4001F EXAM
DIGITAL SIGNAL PROCESSING

University of Cape Town
Department of Electrical Engineering

June 2006
3 hours

Information

- The exam is closed-book.
- There are two parts to this exam.
- **Part A** has *seven* questions totalling 70 marks. You must answer all of them.
- **Part B** has *three* questions, each counting 15 marks. You must answer *two* of them.
- A table of standard z-transform pairs appears at the end of this paper.
- A formula sheet for the radar/sonar question appears at the end of this paper.
- You have 3 hours.

PART A

Answer all of the following questions.

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1. A function $g[n]$ is defined by

$$g[n] = \begin{cases} -2 & n < -4 \\ n & -4 \leq n < 1 \\ \frac{4}{n} & 1 \leq n. \end{cases}$$

Sketch

(a) $y_1[n] = g[n]$

(b) $y_2[n] = g[-n]$

(c) $y_3[n] = g[2-n]$

(d) $y_4[n] = 3g[2n]$

(e) $y_5[n] = (g[n])^2$.

(10 marks)

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2. The transfer function of a causal LTI discrete-time system is given by

$$H(z) = z^{-2}\frac{1 + 1.2z^{-1} + 0.27z^{-2}}{1 + 0.3z^{-1} - 0.18z^{-2}}.$$

You are required to determine the output $y[n]$ of the system for the input

$$x[n] = (0.4)^n u[n] + 6(-0.3)^n u[n].$$

(a) Show that the output satisfies

$$Y(z) = \frac{7(1 + 0.9z^{-1})}{z^2(1 - 0.4z^{-1})(1 + 0.6z^{-1})}$$

and specify the required ROC.

(b) Hence find $y[n]$.

(10 marks)

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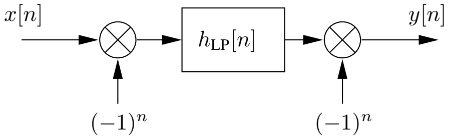
3. If $h_{\mathrm{LP}}[n]$ is the impulse response of a lowpass filter, show that the structure



implements a highpass filter. What is the relationship between the cutoff frequencies of these two filters?

(10 marks)

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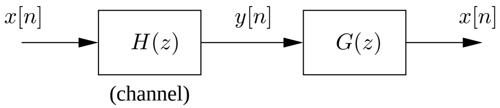
4. A given transmission channel is characterised by the causal transfer function

$$H(z) = \frac{(3z - 2.1)(z^2 - z^1 + 0.25)}{(z - 0.65)(z + 0.48)}.$$

In order to correct for the distortion introduced by the channel on a signal passing through it, we wish to connect a causal stable digital filter characterised by a transfer function $G(z)$ at the receiving end:



Determine $G(z)$ and its region of convergence, and plot the poles and zeros in the z-plane. Justify your answer.

(10 marks)

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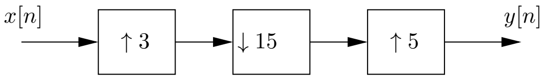
5. Develop an expression for the output $y[n]$ as a function of the input $x[n]$ for the multirate structure below:



(10 marks)

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6. Determine the output response $y[n]$ of a LTI discrete-time system with an impulse response

$$h[n] = \frac{\sin\left((n-2)\frac{\pi}{3}\right)}{(n-2)\pi}$$

for an input

$$x[n] = 3\sin\left(\frac{\pi n}{4}\right) + 5\cos\left(\frac{2\pi n}{5}\right).$$

(10 marks)

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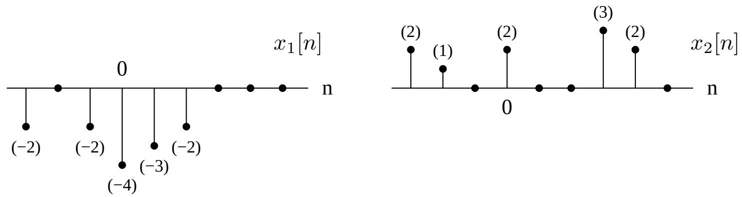
7. Find the N-point circular convolution of the signals $x_1[n]$ and $x_2[n]$ below, for the case of $N = 5$:



Describe how you could calculate this using the discrete Fourier transform.

(10 marks)

PART B

Answer *two* of the following three questions. Each question counts 15 marks.

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1. **Multidimensional signal and image processing**

(a) Discrete-time linear convolution in two dimensions takes the form of the operation

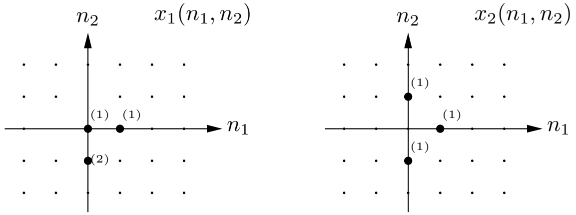
$$y(n_1, n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} h(k_1, k_2) x(n_1 - k_1, n_2 - k_2),$$

where $x(n_1, n_2)$ is an input signal and $h(n_1, n_2)$ a kernel (impulse response) that determines the specific characteristics of the filtering operation being applied.

In practice there are computational advantages if the impulse response is separable, which means that it can be decomposed in the form $h(n_1, n_2) = h_1(n_1)h_2(n_2)$. Explain how this condition can be used to facilitate the convolution, with details as needed. Assume that $h(n_1, n_2)$ is only nonzero for $-5 \leq n_1 < 5$ and $-5 \leq n_2 < 5$.

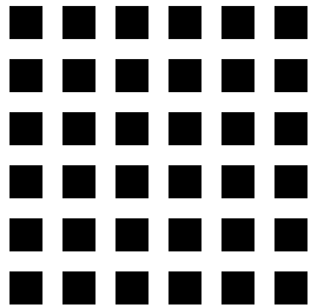
(5 marks)

(b) Use the method of your choice to find the 2-D convolution between the following two signals:



(5 marks)

(c) In the pattern below



the white areas in between the dark squares appear with varying shades of gray. This is an artifact of the human visual system, which can be modelled as linear and shift invariant. Using the notions of impulse responses and/or transfer functions explain how this illusion comes about, and what it tells us about our eyes.

(5 marks)

2. **Speech processing**

Answer **two** of the following three questions.

(a) **FFT-based questions**

i. How many complex multiplications are required to compute an N-point DFT of a complex sequence $x[n]$, given the DFT as $X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}$? (½ mark)

ii. Derive the equation of a radix 5 decimation-in-time algorithm showing the five $N/5$ DFTs and the twiddle factors. (4 marks)

iii. Derive the equation of a radix 2 decimation-in-frequency algorithm showing the two $N/2$ DFTs. (2 marks)

iv. By what factor will a radix 5 decimation-in-time reduce the multiplications and additions compared to a direct computation of the DFT. (1 mark)

(7½ marks)

(b) **Other transforms: LPC and DCT**

i. Which speech sounds are suitable for LPC analysis? (2 marks)

ii. Can an LPC transform be reversible? (1 mark)

iii. Why does the GSM standard (mobile phones) use LPC coding instead of DCT (discrete cosine transforms) or DFT (discrete Fourier transforms). (1½ marks)

iv. Why are there different cosine transforms such as DCT-1, DCT-2, etc? (1 ½ marks)

v. DCT-2 is used mostly in speech processing. Why? (1½ marks)

(7½ marks)

(c) **DFT based question** The logarithm of the magnitude of the DFT of $x[n] = \cos(\frac{2\pi}{8}n)$ for $N = 64$ shows a pulse on $k = 8$ with magnitude of approximately 30.

The logarithm of the magnitude of the DFT of $x[n] = 0.75\cos(\frac{2\pi}{8}n)$ for $N = 64$ shows a pulse at $k = 8$ with magnitude of magnitude 27.

Log magnitude is computed as $20\log_{10}(|X[k]|)$.

i. Given this information plot the log magnitude of the DFT of the following sequences

A. $x_1[n] = \cos(\frac{2\pi}{14}n) + 0.75\cos(\frac{4\pi}{15}n)$. (2 marks)

B. $x_2[n] = \cos(\frac{2\pi}{16}n) + 0.75\cos(\frac{2\pi}{8}n)$. (2 marks)

C. $x_3[n] = \cos(\frac{\pi}{4}n) + 0.001\cos(\frac{21\pi}{64}n)$. (2 marks)

ii. If the DFT analysis was done with $N = 128$, what differences are likely to be observed? (1½ marks)

(7½ marks)

3. **Radar/sonar signal processing**

A high resolution pulsed radar operates at a centre frequency of 10 GHz with a pulse bandwidth of 100 MHz.

(a) Draw a neatly labelled block diagram of a coherent pulsed radar system showing (i) transmitter chain (ii) receiver with an I-Q down converter (iii) appropriate sampling into a digital signal processor. (2 marks)

(b) What is the function of the I-Q down-converter and how is its output related to the *analytic* representation of the received signal? (2 marks)

(c) What is the minimum sample rate required for the I and Q channels? (1 mark)

(d) Draw a neatly labelled block diagram of an equivalent *analytic* signal model of the radar, relating the impulse response of the scene to final output in complex *baseband* form. (2 marks)

(e) Explain briefly the difference between signal processing with a *matched* filter as opposed to a *deconvolution* filter? (4 marks)

(f) Calculate the range resolution of the radar (1 mark)

(g) What property of the transmitted pulse determines the signal to noise ratio after appropriate signal processing (1 mark)

(h) i. If a *deconvolution* filter is used to process the radar data, sketch the expected output (both magnitude and phase) for the case of a "point target" located at a distance of R metres from the antenna. (1 mark)

ii. What effect will the application of a "Window filter" (e.g. Hanning window) have on the processed response? (1 marks)

(15 marks)

Solution not available.

Discrete-time Fourier transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(e^{j\omega})$, $Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n-n_d]$	$e^{-j\omega n_d}X(e^{j\omega})$	Time shift
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Modulation

Common discrete-time Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Z-transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(z)$, $Y(z)$	ROC	Property
$ax[n] + by[n]$	$aX(z) + bY(z)$	ROC contains $R_x \cap R_y$	Linearity
$x[n-n_d]$	$z^{-n_d}X(z)$	ROC $= R_x$	Time shift
$z_0^n x[n]$	$X(z/z_0)$	ROC $= \lvert z_0\rvert R_x$	Frequency scale
$x^*[-n]$	$X^*(1/z^*)$	ROC $= \frac{1}{R_x}$	Time reversal
$nx[n]$	$-z\frac{dX(z)}{dz}$	ROC $= R_x$	Frequency diff.
$x[n] * y[n]$	$X(z)Y(z)$	ROC contains $R_x \cap R_y$	Convolution
$x^*[n]$	$X^*(z^*)$	ROC $= R_x$	Conjugation

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\begin{cases} a^n & 0 \le n \le N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$r^n\cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > \lvert r\rvert$