

EEE401F EXAM

DIGITAL SIGNAL PROCESSING

University of Cape Town
Department of Electrical Engineering

June 2004

3 hours

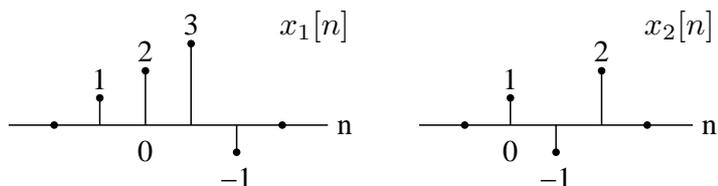
Information

- The exam is closed-book.
 - There are two parts to this exam.
 - **Part A** has *six* questions totalling 70 marks. You must answer all of them.
 - **Part B** has *two* questions totalling 30 marks. You must answer all of them.
 - You have 3 hours.
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PART A

Answer all of the following questions.

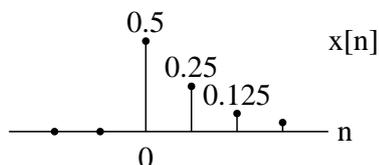
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1. You are given the two signals $x_1[n]$ and $x_2[n]$ shown below. The signal values are zero outside of the range of n shown:



- (a) Use graphical methods to find the linear convolution $y[n] = x_1[n] * x_2[n]$.
(b) Use the expression for the convolution sum directly to find the values of $y[n]$ for $n = -1$ and $n = 1$.

(10 marks)

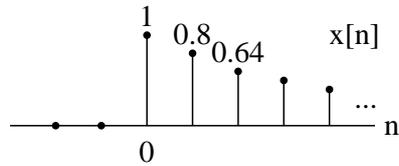
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2. Consider the aperiodic exponentially decaying signal shown below:



- (a) Find the Fourier transform of the $x[n]$ and sketch its magnitude over the range -2π to 2π . Does the signal have predominantly low or high-frequency content?
(b) Calculate the 4-point DFT $X[k]$ of the signal.
(c) If $\tilde{x}[n]$ is the inverse DFT of $X[k]$, what is the value of $\tilde{x}[-2]$ and $\tilde{x}[4]$?

(10 marks)

3. (a) Find the z-transform of the exponentially-decaying signal



- (b) What is the z-transform of the signal $y[n] = x[n] * x[n]$? Use this result, along with the transform pair

$$na^n u[n] \xleftrightarrow{\mathcal{Z}} \frac{az^{-1}}{(1 - az^{-1})^2},$$

to find $y[n]$.

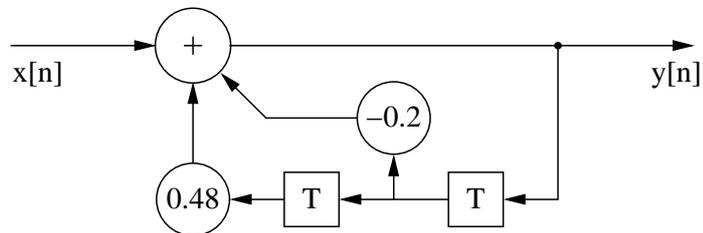
- (c) Use either a power-series expansion or the method of long division to find the first 4 values of the causal signal corresponding to the z-transform

$$X(z) = \frac{1}{1.2 + z}.$$

Where is the region of convergence in the z-plane?

(15 marks)

4. The figure below shows a second-order system:

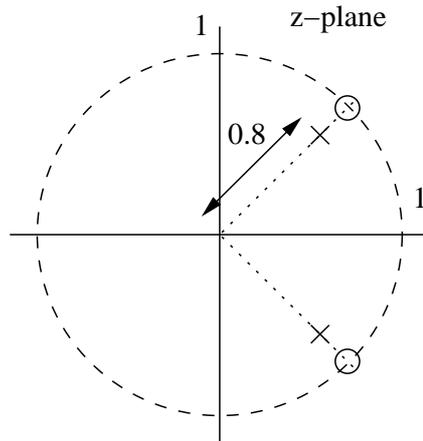


- (a) Write down an expression for the z-transform of the system.
 (b) Under the assumption that the system is causal, draw a pole-zero plot of the system in the z-plane. Is the system stable?
 (c) Find the impulse response of the system under the assumptions given. You may need the transform pair

$$a^n u[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{1 - az^{-1}}, \quad |z| > |a|.$$

(15 marks)

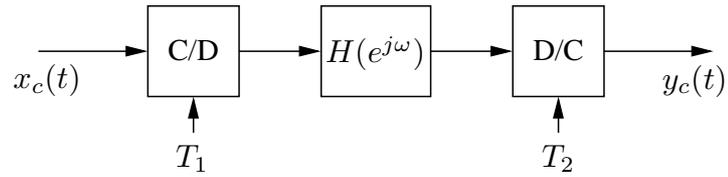
5. The pole-zero plot for a filter is as follows:



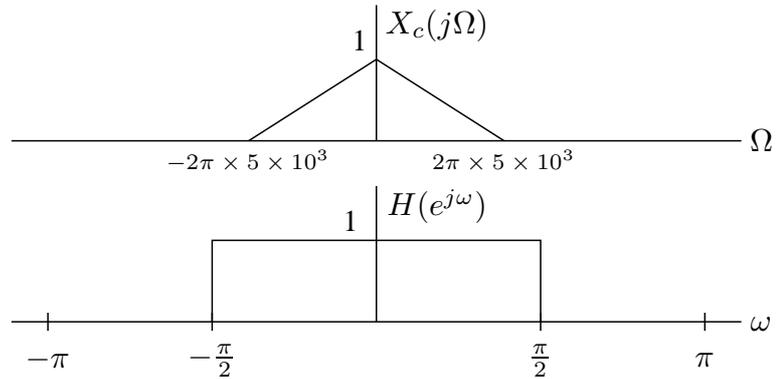
- Use graphical methods to roughly sketch the magnitude and phase response of the filter.
- Describe the effect the filter will have on an input signal.
- What are the possible regions of convergence for the filter? Which will correspond to a causal filter, and which to a stable filter?

(10 marks)

6. In the system



$X_c(j\Omega)$ and $H(e^{j\omega})$ are as shown below:



Sketch and label the Fourier transform of $y_c(t)$ for each of the following cases:

- (a) $1/T_1 = 1/T_2 = 10^4$.
- (b) $1/T_1 = 1/T_2 = 2 \times 10^4$.
- (c) $1/T_1 = 2 \times 10^4, 1/T_2 = 10^4$.
- (d) $1/T_1 = 10^4, 1/T_2 = 2 \times 10^4$.

(10 marks)

PART B

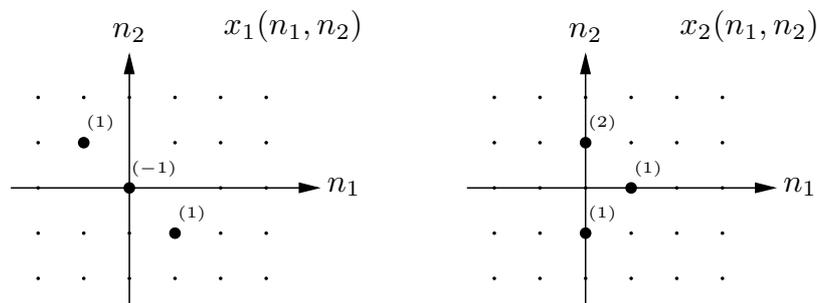
Answer *both* of the following two questions. Each question counts 15 marks.

1. Multidimensional signal and image processing

- (a) Explain why a periodic sequence in 2-D is more complex than for 1-D.

(5 marks)

- (b) Use the method of your choice to find the 2-D convolution between the following two signals:



(5 marks)

- (c) Describe in detail how a charge-coupled device (CCD) creates a signal.

(5 marks)

2. Speech processing

(a) Radix-3 Decimation in time FFT algorithm:

- i. Write an expression for $X[k]$ involving three $N/3$ -DFTs.
- ii. What is the advantage of a radix-3 FFT for a $N = 3^p$ length signal as compared to a direct computation of the DFT?

(5 marks)

(b) True or False:

- i. A radix-2 Decimation-in-time FFT algorithm will be faster than a radix-2 Decimation-in-frequency FFT algorithm.
- ii. A spectrogram contains 3-D information about a speech signal.
- iii. Estimating nasals parameters is one of the weakness of LPC model.
- iv. Zero padding a signal before computing its DFT will increase its frequency resolutions.
- v. Vowels have distinct formants.

(5 marks)

(c) The following are three different signals $x_i[n]$ that are the sum of two sinusoids:

$$x_1[n] = \cos(\pi n/4) + \cos(17\pi n/64)$$

$$x_2[n] = \cos(\pi n/4) + 0.8 \cos(21\pi n/64)$$

$$x_3[n] = \cos(\pi n/4) + 0.0001 \cos(21\pi n/64)$$

We wish to estimate the spectrum of each of these signals using the 64-point DFT with a 64-point rectangular window $w[n]$. Indicate for which of the signals you would expect to see two distinct spectral peaks.

(5 marks)
