

EEE4001F: Digital Signal Processing

Class Test 1

20 March 2013

Name:

Student number:

Information

- The test is closed-book.
 - This test has *four* questions, totalling 20 marks.
 - Answer *all* the questions.
 - You have 45 minutes.
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1. (5 marks) Given the sequence

$$x[n] = 2\delta[n + 3] + (3 - n)(u[n] - u[n - 3]),$$

sketch the following sequences (for $-4 \leq n \leq 4$):

(a) $y_1[n] = x[n]$

(b) $y_2[n] = x[2n - 3]$

(c) $y_3[n] = x[|n|]$.

2. (5 marks) A linear time-invariant system has an impulse response given by $h[n] = a^{-n}u[-n]$, $0 < a < 1$, where $u[n]$ is the unit step sequence

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0. \end{cases}$$

Determine the response to the input $x[n] = u[n]$.

3. (5 marks) Consider two discrete-time LTI systems which are characterized by their impulse responses $h_1[n] = \delta[n] - \delta[n - 1]$ and $h_2[n] = u[n]$.
- (a) Determine whether these two LTI systems are inverses of each other. Justify your answer.
 - (b) Determine whether these systems are stable, memory-less, and causal. Justify your answer.

4. (5 marks) An LTI system is described by the input-output relation

$$y[n] = x[n] + 2x[n - 1] + x[n - 2].$$

- (a) Determine the impulse response $h[n]$
- (b) Is this a stable system?
- (c) Show that the frequency response of the system can be written as

$$H(e^{j\omega}) = 2e^{-j\omega}(\cos(\omega) + 1).$$

- (d) Plot the magnitude and phase of $H(e^{j\omega})$
- (e) Now consider a new system whose frequency response is $H_1(e^{j\omega}) = H(e^{j(\omega+\pi)})$. Determine $h_1[n]$, the impulse response of the new system.

Fourier transform properties

Sequences $x[n], y[n]$	Transforms $X(e^{j\omega}), Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
1 $(-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n]$ $(a < 1)$	$\frac{1}{1 - ae^{-j\omega}}$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n + 1)a^n u[n]$ $(a < 1)$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \omega < \omega_c \\ 0 & \omega_c < \omega \leq \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z < 1$
$\delta[n - m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z > a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z < a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z > a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z < a $
$\begin{cases} a^n & 0 \leq n \leq N - 1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$ z > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2 z^{-2}}$	$ z > r$