EEE4001F: Digital Signal Processing

Class Test 1

17 March 2011

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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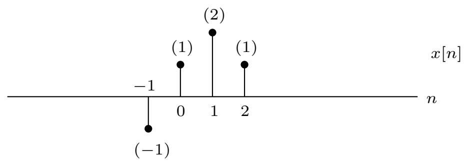
1. (5 marks) If $x[n]$ is the signal below



then plot the following:

(a) $y_1[n] = x[2n-2]$

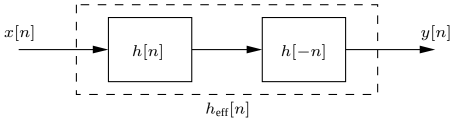
(b) $y_2[n] = x[-n-1]$

(c) $y_3[n] = x[n] * \delta[n-1]$

(d) $y_4[n] = x[n-1] * u[n-1]$

(e) $y_5[n] = x[n] - x[n-1]$.

2. (5 marks) Consider the system below



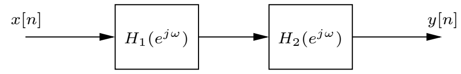
where $h[n] = \delta[n] - 2\delta[n-1] - \delta[n-2]$.

(a) Find $H(e^{j\omega})$, the Fourier transform of $h[n]$.

(b) Find and plot the effective impulse response $h_{\text{eff}}[n]$ linking the input $x[n]$ and the output $y[n]$.

(c) Give an expression for the effective system transfer function $H_{\text{eff}}(e^{j\omega})$ in terms of $H(e^{j\omega})$.

3. (5 marks) Suppose we have the following cascade of all-pole filters:



where

$$H_1(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} \qquad \text{and} \qquad H_2(e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}.$$

(a) Find $h[n]$ such that the input $x[n]$ and output $y[n]$ satisfy the relationship $y[n] = h[n] * x[n]$.

(b) Find $g[n]$ such that $x[n] = g[n] * y[n]$.

4. (5 marks) Consider the minimum-phase system with transfer function

$$H(z) = \frac{1-(1/2)z^{-1}}{1-(1/3)z^{-1}},$$

where the region of convergence is $|z| > \frac{1}{3}$.

(a) Find a difference equation linking the input $x[n]$ and the output $y[n]$.

(b) Find the impulse response of the system.

(c) Find the transfer function of a causal and stable system that is the inverse of $H(z)$, and sketch the poles, zeros, and region of convergence of this inverse system in the z-plane.

Fourier transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(e^{j\omega})$, $Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n-n_d]$	$e^{-j\omega n_d}X(e^{j\omega})$	Time shift
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \|\omega\| < \omega_c \\ 0 & \omega_c < \|\omega\| \leq \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$\begin{cases} a^n & 0 \leq n \leq N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$r^n\cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$