EEE4001F: Digital Signal Processing

Class Test 1

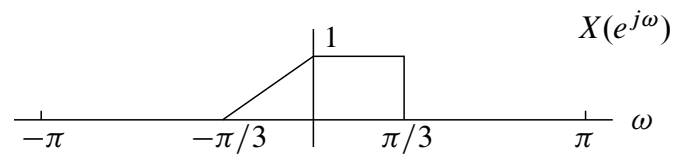### 20 March 2008

Name:

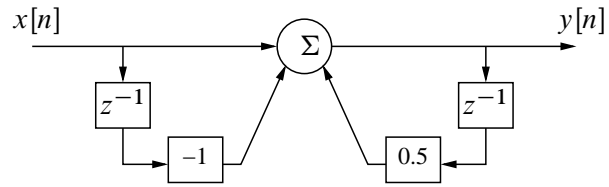**Student number:**

Information

- The test is closed-book.

- This test has *four* questions, totalling 20 marks.

- Answer *all* the questions.

- You have 45 minutes.

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1. (5 marks) A sequence $x[n]$ has a zero-phase DTFT $X(e^{j\omega})$ given below:

$X(e^{j\omega})$

1

$-\pi$ $-\pi/3$ $\pi/3$ π ω

Sketch the DTFT of the sequence $y[n] = x[-n]e^{-j\pi n/3}$.

2. (5 marks) Consider the following LTI system:

Determine a closed-form expression for the response $y[n]$ of this system to the following input signal:

$$x[n] = \begin{cases} 1 & n \geq 4 \\ 0 & \text{otherwise} \end{cases}$$

if the system is causal and initially at rest.

3. (5 marks) Consider the following discrete-time signal $x[n]$:

$$x[n] = \begin{cases} n + 1 & 0 \le n \le 3 \\ 4 & n \ge 4 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the z-transform $X(z)$ of $x[n]$, and represent it as a ratio of polynomials in z^{-1}.

(b) What is the region of convergence (ROC) of this z-transform?

4. (5 marks)

Consider the continuous-time signal

$$x(t) = \sin(400\pi t + \pi).$$

The discrete-time signal $x[n]$ is obtained by sampling $x(t)$ at $t = n/f_s$ with a sampling frequency $f_s = 1000$ Hz. Which one of the following continuous-time signals will yield the same sample values when sampled at the same sampling instants? Show and motivate your calculations.

(a) $\sin(600\pi t)$

(b) $-\sin(1000\pi t)$

(c) $\sin(1400\pi t)$

(d) $\sin(1600\pi t)$

Fourier transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(e^{j\omega})$, $Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (|a| < 1)$	$\frac{1}{1 - ae^{-j\omega}}$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n + 1)a^n u[n] \quad (|a| < 1)$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$|z| > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$|z| < 1$
$\delta[n - m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$|z| > |a|$
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$|z| < |a|$
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$|z| > |a|$
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$|z| < |a|$
$\begin{cases} a^n & 0 \le n \le N - 1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$|z| > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$|z| > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2 z^{-2}}$	$|z| > r$