EEE4001F: Digital Signal Processing

Class Test 1

22 March 2007

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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1. (5 marks) Determine the impulse response of the LTI system described by the difference equation

$$y[n] - 0.2y[n-1] = x[n] + 0.5x[n-1]$$

under the assumption that it is causal. Is the system stable?

The z-transform of the difference equation is

$$Y(z)(1 - 0.2z^{-1}) = X(z)(1 + 0.5z^{-1})$$

so

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 0.5z^{-1}}{1 - 0.2z^{-1}} = \frac{z + 0.5}{z - 0.2}$$

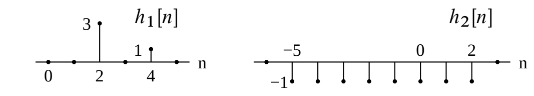
and the system has a pole at $z = 0.2$. The ROC for a causal system wil be outside this pole: $|z| > 0.2$. This ROC includes the unit circle, so $h[n]$ has a Fourier transform, is absolutely summable, and therefore corresponds to a stable system.

2. (5 marks) Which of the impulse responses

$$h_1[n] = 3\delta[n-2] + \delta[n-4]$$
$$h_2[n] = u[n-3] - u[n+5]$$

describe causal, stable, LTI processors? Give reasons for your answers. Sketch the step response of each system.

The impulse responses are



Since $h_1[n] = 0$ for $n < 0$, it is the impulse response of a causal system. The system described by $h_2[n]$ is not causal: the output when the input is a unit impulse at the origin starts at $n = -5$.

Both systems are stable, since their impulse responses are absolutely summable:

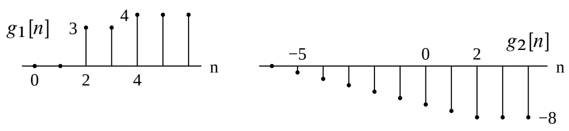
$$S_1 = \sum_{n=-\infty}^{\infty} |h_1[n]| = 4 \quad \text{and} \quad S_2 = \sum_{n=-\infty}^{\infty} |h_2[n]| = 8.$$

Thus both systems are stable.

The step response is the convolution of the impulse response with the unit step:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]u[n-k] = \sum_{k=-\infty}^{n} h[k]u[n-k] = \sum_{k=-\infty}^{n} h[k],$$

which is just the accumulated signal. Thus the respective step responses are



3. (5 marks) Convolve the signals

$$x_1[n] = \delta[n] - \delta[n-2] + \delta[n-3] \quad \text{and} \quad x_2[n] = 2\delta[n-1] + \delta[n-2] - \delta[n-3]$$

using the z-transform.

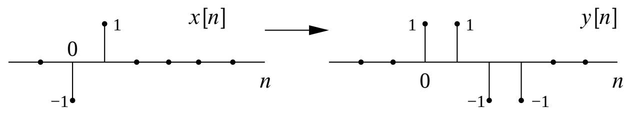
The signals in the z-domain are $X_1(z) = 1 - z^{-2} + z^{-3}$ and $X_2(z) = 2z^{-1} + z^{-2} - z^{-3}$. The convolution is the product

$$\begin{aligned} Y(z) = X_1(z)X_2(z) &= [1 - z^{-2} + z^{-3}][2z^{-1} + z^{-2} - z^{-3}] \\ &= 2z^{-1} + z^{-2} - 3z^{-3} + z^{-4} + 2z^{-5} - z^{-6}, \end{aligned}$$

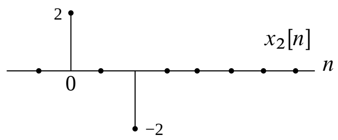
which inverts to

$$y[n] = 2\delta[n-1] + \delta[n-2] - 3\delta[n-3] + \delta[n-4] + 2\delta[n-5] - \delta[n-6].$$

4. (5 marks) Suppose $y[n]$ is the output of an LTI system when $x[n]$ is the input:



Find the response of the system to the input



The response to $x[n]$ is $y[n]$. Since the system is LTI, the response to $2x[n]$ is $2y[n]$ and the response to $-2x[n-1]$ is $-2y[n-1]$. But observe that the signal $x_2[n]$ is just $2x[n] - 2x[n-1]$, so the output must be the sum $2y[n] - 2y[n-1]$.