EEE4001F: Digital Signal Processing

Class Test 1

27 March 2006

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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1. (5 marks) Determine the impulse response of the LTI system described by the difference equation

$$y[n] - 0.35y[n-1] = x[n]$$

under the assumption that it is (a) causal and (b) not causal.

The impulse response is the output $y[n] = h[n]$ of the system when the input is $x[n] = \delta[n]$. If the system is causal, then for this input the output must be zero for $n < 0$, so we must have $h[-1] = 0$. Iterating

$$h[n] = 0.35h[n-1] + \delta[n]$$

in the forward direction gives the values of the impulse response

$$\begin{aligned} h[0] &= 1 \\ h[1] &= 0.35 \\ h[2] &= (0.35)h[1] = 0.35^2 \\ h[3] &= (0.35)h[2] = 0.35^3 \end{aligned}$$

and so on. The general solution is $h[n] = 0.35^n u[n]$.

There is a second noncausal (anticausal) impulse response corresponding to the reverse iteration

$$h[n-1] = (0.35)^{-1}(h[n] - \delta[n])$$

with the initial condition $h[1] = 0$ (this is clear with the hindsight of the z-transform and its regions of convergence, but subtle arguments are required for determining this in the time domain (see O&S p.37, for example). In any case, this leads to the values of the impulse response

$$\begin{aligned} h[0] &= 0 \\ h[-1] &= -(0.35) \\ h[-2] &= (0.35)h[1] = -(0.35)^2 \\ h[-3] &= (0.35)h[2] = -(0.35)^3, \end{aligned}$$

or $h[n] = -(0.35)^{-n}u[-n-1]$ in general.

2. (5 marks) Sketch the sequence

$$y[n] = \alpha^{|n|}$$

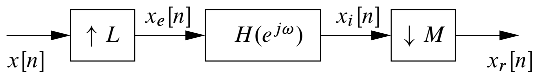
for $|\alpha| < 1$ and find its DTFT. Why do we require $|\alpha| < 1$?

The required transform is

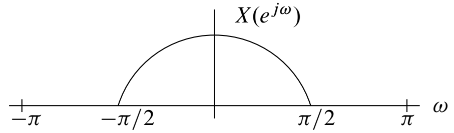
$$\begin{aligned}
Y(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} \alpha^{|n|} e^{-j\omega n} \\
&= \sum_{n=-\infty}^{0} \alpha^{|n|} e^{-j\omega n} + \sum_{n=0}^{\infty} \alpha^{|n|} e^{-j\omega n} - \alpha^{|0|} e^{0} \\
&= \sum_{n=-\infty}^{0} \alpha^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} \alpha^{n} e^{-j\omega n} - 1 \\
&= \sum_{n=0}^{\infty} \alpha^{n} e^{j\omega n} + \sum_{n=0}^{\infty} \alpha^{n} e^{-j\omega n} - 1 \\
&= \sum_{n=0}^{\infty} (\alpha e^{j\omega})^{n} + \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^{n} - 1 \\
&= \frac{1}{(1 - \alpha e^{j\omega})} + \frac{1}{(1 - \alpha e^{-j\omega})} - 1
\end{aligned}$$

since each infinite sums exists for $|\alpha| < 1$.

3. (5 marks) Describe how a structure of the form

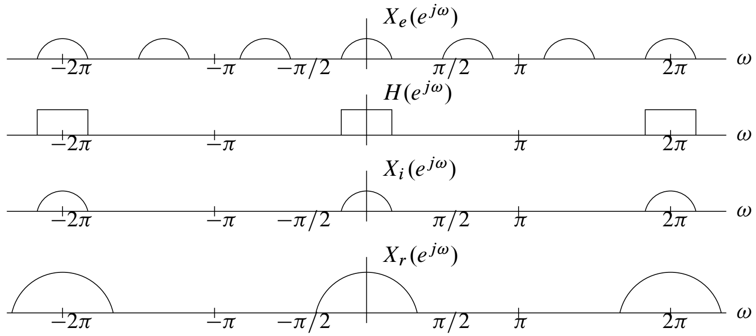


can be used to increase the sampling rate of the signal $x[n]$ by a factor of 1.5. Sketch representative Fourier transforms of the signals at different points in the system if



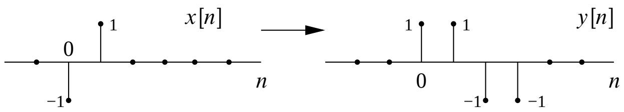
and specify $H(e^{j\omega})$.

To increase the sampling rate by the required factor we need to expand the signal by a factor of 3 ($L = 3$), filter to eliminate undesired images, and decimate by a factor of 2 ($M = 2$). Since the overall rate is increased, there is no danger of aliasing. The signals in the system are as follows:

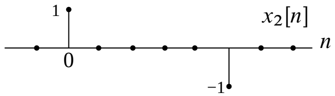


The lowpass filter $H(e^{j\omega})$ as drawn has a cutoff frequency of $\pi/6$, but any cutoff in the range $\pi/6$ to $\pi/2$ will meet the filtering requirements.

4. (5 marks) Suppose $y[n]$ is the output of an LTI system when $x[n]$ is the input:



(a) What is the response of the system to the input



(b) Find the impulse response $h[n]$ of this system.

Starting with the impulse response, we know that

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[0]h[n-0] + x[1]h[n-1] = -h[n] + h[n-1].$$

If the system is causal then

$$\begin{aligned}
y[0] = 1 = -h[0] + h[-1] = -h[0] &\Longrightarrow h[0] = -1 \\
y[1] = 1 = -h[1] + h[0] = -h[1] - 1 &\Longrightarrow h[1] = -2 \\
y[2] = -1 = -h[2] + h[1] = -h[2] - 2 &\Longrightarrow h[2] = -1 \\
y[3] = -1 = -h[3] + h[2] = -h[3] - 1 &\Longrightarrow h[3] = 0
\end{aligned}$$

and so on. The impulse response is therefore

$$h[n] = \begin{cases} -1 & n = 0 \\ -2 & n = 1 \\ -1 & n = 2 \\ 0 & \text{otherwise} \end{cases}$$

Using linearity and time invariance the response to $x_2[n] = \delta[n] - \delta[n-5]$ is simply $y_2[n] = h[n] - h[n-5]$, with $h[n]$ given above.