# EEE4001F EXAM
# DIGITAL SIGNAL PROCESSING

University of Cape Town
Department of Electrical Engineering

June 2007
3 hours

## Information

- The exam is closed-book.
- There are two parts to this exam.
- **Part A** has *seven* questions totalling 70 marks. You must answer all of them.
- **Part B** has *two* questions, each counting 15 marks. You must answer *both* of them.
- A table of standard z-transform pairs appears at the end of this paper.
- A formula sheet for the radar/sonar question appears at the end of this paper.
- You have 3 hours.



# PART A

Answer all of the following questions.

1. Given
$$x[n] = 4\delta[n-1] + 2\delta[n-2] + \delta[n-3],$$
sketch
   (a) $x_1[n] = x[n]$.
   (b) $x_2[n] = x[2n]$.
   (c) $x_3[n] = x[2-n]$.
   (d) $x_4[n] = \sum_{k=-\infty}^{n} x[k]$.
   (e) $x_5[n] = x[n] - x[n-1]$.

(10 marks)

2. Determine a closed-form expression for the impulse response of the causal LTI system described by the difference equation
$$y[n] - 0.3y[n-1] - 0.04y[n-2] = x[n] + 2x[n-1].$$
Is the system stable? Why?

(10 marks)

3. Find the DTFTs of the following functions:
   (a) $x_1[n] = \text{rect}_2[n]$
   (b) $x_2[n] = \text{rect}_2[n] * 3\delta[n+3]$
   (c) $x_3[n] = \text{rect}_2[n] * (-5\text{rect}_2[n])$,

   where $\text{rect}_2[n]$ is defined as
$$\text{rect}_2[n] = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

(10 marks)

4. A digital filter is designed by cascading two identical FIR filters with impulse response

$$h[n] = \begin{cases} n+1 & 0 \le n \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the impulse response $g[n]$ of the composite filter by means of linear convolution.

(10 marks)

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5. A feedback discrete-time system has a transfer function

$$H(z) = \frac{K}{1 + K(z/(z-0.9))}.$$

For what range of real values of $K$ is this system stable?

(10 marks)

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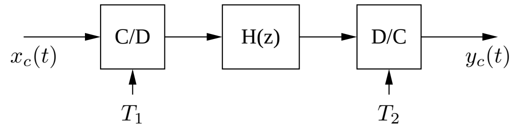
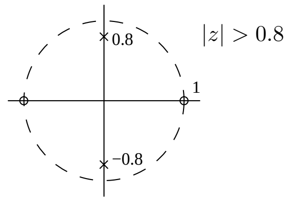
6. The continuous-time filter




uses a discrete-time processor with the system function $H(z)$ shown below:




(a) Sketch the magnitude response of the discrete-time filter $H(z)$ over the range $0 \le \omega \le 2\pi$.

(b) Assuming that the filter has unity gain at the center of its passband, specify an algebraic form for the system function $H(z)$.

(c) Assuming that $T_1 = 10^{-4}$ and $T_2 = 2 \times 10^{-4}$, find the steady-state response of the overall system to the input

$$x_c(t) = \cos(2\pi(2500)t).$$

(10 marks)

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7. When using DSP design software, filters can be defined parametrically. Assuming we are interested in designing a lowpass filter, sketch the generic lowpass filter characteristic and state the parameters that the user must specify. Discuss the ”cost” implications of these parameters in terms of engineering tradeoffs.

(10 marks)

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# PART B

Answer *both* of the following questions. Each question counts 15 marks.

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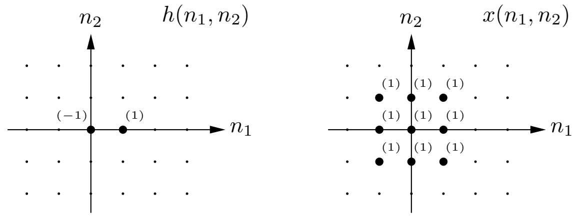
1. **Image processing and computer vision**

   (a) Find and sketch the 2-D convolution between the following two signals:




(5 marks)

   (b) The signal $h(n_1, n_2)$ in the previous question is the impulse response of a 2-D edge detector. What edge direction does it detect? Draw the impulse response of a filter that can be used to detect edges in the orthogonal direction. How could you modify these filters to reduce noise in the filter output?

(5 marks)

   (c) In some application you observe 4 data points with the following $(x, y)$ coordinates: $(1, 1.4)$, $(1.9, 1.9)$, $(3, 0.9)$, $(3.2, 1.8)$. You need to fit a straight line with zero slope to these points: the parametric family of curves is therefore

$$f_\theta(x) = \theta,$$

   where $\theta$ is the parameter to be determined. Formulate this problem as a least-squares minimisation and find the optimum value of the parameter. What is the least-squares value at the minimum?

(5 marks)



2. **Radar/sonar signal processing**

(a) Draw a neatly labelled block diagram of a radar showing an I-Q down converter in the receiving chain. (2 marks)

(b) The radar transmits a chirp pulse of bandwidth $B = 10$ MHz, centred on some centre frequency $f_0$ Hz. Using a series of labelled sketches, illustrate how the *frequency spectrum* $V_{bb}(f)$ of the baseband complex signal obtained from the I-Q down-converter relates to (i) the transmitted pulse $V_{tx}(f)$ (ii) the sprectrum $\zeta(f)$ of the impulse response of the scene. (2 marks)

(c) What sample rate is required for the analogue to digital converters sampling the I and Q channels. (1 mark)

(d) If the scene contains two "point" targets at ranges 1000m and 1100m, write down an expression which models the impulse response of the scene. (1 marks)

(e) Assuming an *inverse (deconvolution)* filter is used to process the received echo from the two targets, sketch the magnitude response that you would expect to see at the output of the filter (as a function of time). NOTE: You need only show relative amplitudes in your sketch. What is the 3dB range resolution? How could you improve the reduce the sidelobe levels of the response? (3 marks)

(f) In the design of a high resolution radar, what properties of the transmitted pulse are important to consider in its design? Why a chirp pulse is preferred to a monochrome pulse for such a radar. (3 marks)

(g) Explain briefly the differences between a *matched* filter and an *inverse* (*deconvolution)* filter. You can assume that the receiver has a perfect bandpass filter.
(i) Sketch the magnitude spectrum of the two types of filters for the case of a monochrome pulse.
(ii) Sketch the magnitude spectrum of the two types of filters for the case of a a chirp pulse with a large time bandwidth product.
(3 marks)

## Fourier transform properties

| Sequences $x[n], y[n]$ | Transforms $X(e^{j\omega}), Y(e^{j\omega})$ | Property |
|---|---|---|
| $ax[n] + by[n]$ | $aX(e^{j\omega}) + bY(e^{j\omega})$ | Linearity |
| $x[n - n_d]$ | $e^{-j\omega n_d} X(e^{j\omega})$ | Time shift |
| $e^{j\omega_0 n} x[n]$ | $X(e^{j(\omega-\omega_0)})$ | Frequency shift |
| $x[-n]$ | $X(e^{-j\omega})$ | Time reversal |
| $nx[n]$ | $j\frac{dX(e^{j\omega})}{d\omega}$ | Frequency diff. |
| $x[n] * y[n]$ | $X(e^{-j\omega})Y(e^{-j\omega})$ | Convolution |
| $x[n]y[n]$ | $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$ | Modulation |

## Common Fourier transform pairs

| Sequence | Fourier transform |
|---|---|
| $\delta[n]$ | $1$ |
| $\delta[n - n_0]$ | $e^{-j\omega n_0}$ |
| $1 \quad (-\infty < n < \infty)$ | $\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$ |
| $a^n u[n] \quad (\|a\| < 1)$ | $\frac{1}{1-ae^{-j\omega}}$ |
| $u[n]$ | $\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$ |
| $(n+1)a^n u[n] \quad (\|a\| < 1)$ | $\frac{1}{(1-ae^{-j\omega})^2}$ |
| $\frac{\sin(\omega_c n)}{\pi n}$ | $X(e^{j\omega}) = \begin{cases} 1 & \|\omega\| < \omega_c \\ 0 & \omega_c < \|\omega\| \leq \pi \end{cases}$ |
| $x[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$ | $\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$ |
| $e^{j\omega_0 n}$ | $\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$ |

## Common z-transform pairs

| Sequence | Transform | ROC |
|---|---|---|
| $\delta[n]$ | $1$ | All $z$ |
| $u[n]$ | $\frac{1}{1-z^{-1}}$ | $\|z\| > 1$ |
| $-u[-n-1]$ | $\frac{1}{1-z^{-1}}$ | $\|z\| < 1$ |
| $\delta[n-m]$ | $z^{-m}$ | All $z$ except 0 or $\infty$ |
| $a^n u[n]$ | $\frac{1}{1-az^{-1}}$ | $\|z\| > \|a\|$ |
| $-a^n u[-n-1]$ | $\frac{1}{1-az^{-1}}$ | $\|z\| < \|a\|$ |
| $na^n u[n]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\|z\| > \|a\|$ |
| $-na^n u[-n-1]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\|z\| < \|a\|$ |
| $\begin{cases} a^n & 0 \leq n \leq N-1, \\ 0 & \text{otherwise} \end{cases}$ | $\frac{1-a^N z^{-N}}{1-az^{-1}}$ | $\|z\| > 0$ |
| $\cos(\omega_0 n)u[n]$ | $\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$ | $\|z\| > 1$ |
| $r^n \cos(\omega_0 n)u[n]$ | $\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$ | $\|z\| > r$ |