EEE4114F: Digital Signal Processing

Class Test

7 March 2019

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 60 minutes.
- An information sheet is attached.

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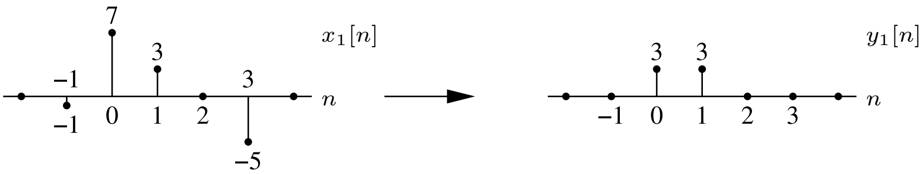
1. (5 marks) The output $y[n]$ of a median filter is given by

$$y[n] = \text{med}(x[n-1], x[n], x[n+1]).$$

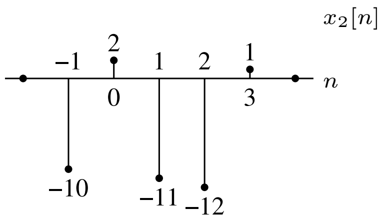
Here the median function, med( ), lists the 3 samples in descending order and selects the value in the middle of the list. For example, the input $x_1[n]$ generates the output $y_1[n]$ below:



(a) Is the system causal? Why?

(b) Is the system stable? Why?

(c) Comment on whether the above median filter is linear. Justify your comments and construct a simple relevant example to illustrate your conclusion. For this purpose you may want to use the example pair given above and the input sequence below:



2. (5 marks) Let $x[n]$ an $v[n]$ be defined as follows:

$$x[n] = \delta[n-1] - \delta[n+1] \qquad \text{and} \qquad v[n] = a\delta[n] + b\delta[n-1] + c\delta[n-2].$$

Suppose now that $y[n] = v[n] * x[n]$ gives

$$y[n] = -\delta[n+1] - 2\delta[n] + 2\delta[n-2] + \delta[n-3].$$

(a) Sketch the signals involved.

(b) Specify values for a, b, and c.

3. (5 marks) Consider the discrete LTI system represented by

$$y[n] = x[n] - x[n-1]$$

where $x[n]$ and $y[n]$ are the input and output respectively.

(a) Determine and plot the impulse response $h[n]$. Is the system stable?

(b) Determine and plot the step response corresponding to $x[n] = u[n]$.

(c) Find $H(e^{j\omega})$ and plot its magnitude.

(d) Determine and plot the response to the input $x[n] = (-1)^n$.

4. (5 marks) A LTI system has a step response of

$$g[n] = n\left(\frac{1}{2}\right)^n u[n].$$

In other words, when the input is $x[n] = u[n]$ then the output is $y[n] = g[n]$ above.

(a) Is the system causal? Why?

(b) Find the Fourier transform of $g[n]$.

(c) Find an expression for its impulse response. You may want to use the fact that $u[n] - u[n-1] = \delta[n]$.

Discrete-time Fourier transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(e^{j\omega})$, $Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n-n_d]$	$e^{-j\omega n_d}X(e^{j\omega})$	Time shift
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Modulation

Common discrete-time Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Z-transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(z)$, $Y(z)$	ROC	Property
$ax[n] + by[n]$	$aX(z) + bY(z)$	ROC contains $R_x \cap R_y$	Linearity
$x[n-n_d]$	$z^{-n_d}X(z)$	ROC $= R_x$	Time shift
$z_0^n x[n]$	$X(z/z_0)$	ROC $= \lvert z_0\rvert R_x$	Frequency scale
$x^*[-n]$	$X^*(1/z^*)$	ROC $= \frac{1}{R_x}$	Time reversal
$nx[n]$	$-z\frac{dX(z)}{dz}$	ROC $= R_x$	Frequency diff.
$x[n] * y[n]$	$X(z)Y(z)$	ROC contains $R_x \cap R_y$	Convolution
$x^*[n]$	$X^*(z^*)$	ROC $= R_x$	Conjugation

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\begin{cases} a^n & 0 \le n \le N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2 z^{-2}}$	$\lvert z\rvert > \lvert r\rvert$