EEE4001F: Digital Signal Processing

Class Test 2

23 April 2009

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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1. (5 marks) Consider the discrete-time signal

$$x[n] = \begin{cases} -1 & n = 0 \\ 2 & n = 1 \\ 2 & n = 2 \\ 1 & n = 3 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the 4-point DFT $X[k]$ of $x[n]$.

Since

$$X[k] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}kn} = \sum_{n=0}^{3} x[n] e^{-j\frac{\pi}{2}kn}$$

we have

$$X[0] = -1e^0 + 2e^0 + 2e^0 + 1e^0 = 4,$$
$$X[1] = -1e^0 + 2e^{-j\pi/2} + 2e^{-j\pi} + 1e^{-j3\pi/2} = -1 - 2j - 2 + j = -3 - j,$$
$$X[2] = -1e^0 + 2e^{-j\pi} + 2e^{-j2\pi} + 1e^{-j3\pi} = -1 - 2 + 2 - 1 = -2,$$
$$X[3] = -1e^0 + 2e^{-j3\pi/2} + 2e^{-j6\pi/2} + 1e^{-j9\pi/2} = -1 + 2j - 2 - j = -3 + j.$$

2. (5 marks) A linear time-invariant filter has the following transfer function:

$$H(z) = 3 + 5z^{-1} - 4z^{-2} + 5z^{-3} - 7z^{-4}.$$

   (a) Find the impulse response of the filter.
   (b) Does the filter have a finite or an infinite impulse response? Why?
   (c) Does the filter have a linear phase response? Why?

   (a) Since $H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$ we can see by equating terms that the impulse response is $h[n] = 3\delta[n] + 5\delta[n-1] - 4\delta[n-2] + 5\delta[n-3] - 7\delta[n-4]$.
   (b) No poles, so FIR.
   (c) For linear phase we require a symmetric or anti-symmetric impulse response around the midpoint. For a 5-point IR, the symmetry would have to be around $n = 2$. since $h[0] \neq h[4]$, the filter is not linear phase.

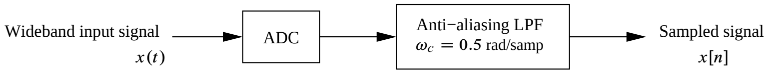
3. (5 marks) A system has the impulse response $h[n] = \alpha\delta[n] + \beta\delta[n-1]$. What conditions are required on α and β for it to have a causal and stable inverse?

   The system has the z-transform $H(z) = \alpha + \beta z^{-1}$, with ROC the entire z-plane excluding the origin. The inverse system has z-transform

$$G(z) = \frac{1}{\alpha + \beta z^{-1}}$$

   which has a pole at $z = -\beta/\alpha$. For a causal inverse the ROC must be taken outside this pole: $|z| > |-\beta/\alpha|$, or $|z| > |\beta/\alpha|$. This inverse is only stable if the ROC includes the unit circle, in which case $|\beta/\alpha| < 1$.

4. (5 marks) Consider the following sampling system:



Explain, with clear motivation, what is wrong with this configuration. What consequences will this fault have?

The anti-aliasing filter must be placed before the ADC, and must hence be an analog filter. For the given configuration, aliasing will occur at the ADC, and the discrete-time "anti-aliasing" filter will not have the desired effect.

[Any elaboration on the above answer will be fine as a solution.]

Fourier transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(e^{j\omega})$, $Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\begin{cases} a^n & 0 \le n \le N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$