EEE401F: Digital Signal Processing

Class Test 1

18 March 2004

Name:

Student number:

Information

- The test is closed-book.

- This test has *five* questions, totalling 25 marks.

- Answer *all* the questions.

- You have 45 minutes.

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1. (5 marks) Find and sketch the unit step response of the causal LTI processor defined by the following recurrence formula:

$$y[n] = -0.5y[n-1] + x[n]$$

Is the resulting sequence stable?

2. (5 marks) Find the impulse response of an overall system formed by cascading two LTI processors with the impulse responses:

$$h_1[n] = \begin{cases} \frac{1}{n} & (0 < n < 4) \\ 0 & \text{otherwise} \end{cases}$$

and

$$h_2[n] = \begin{cases} n & (0 < n < 4) \\ 0 & \text{otherwise.} \end{cases}$$

3. (5 marks) Find a nonrecursive recurrence formula which, from the DSP point of view, is equivalent to the following recursive formula for a causal filter:
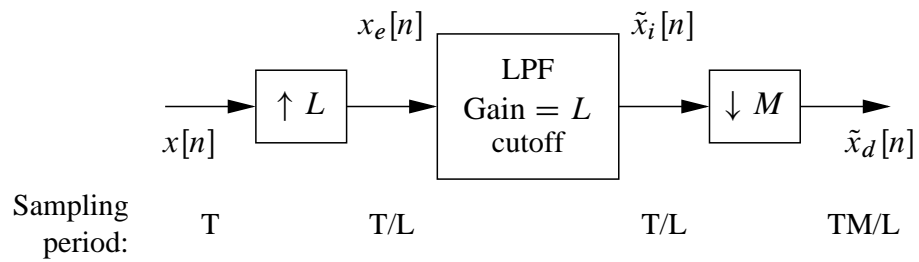
$$y[n] = y[n-1] + x[n] - x[n-7].$$

What is the relative computational economy of the recursive and nonrecirsive versions?

4. (5 marks) A signal has the z-transform

$$X(z) = \frac{1}{z(z-1)(2z-1)},$$

with region of convergence $|z| > 1$. Draw a pole-zero plot of the signal in the z-plane, and use the method of partial fractions to recover the signal $x[n]$. Is the signal stable? Is the signal causal?

5. (5 marks) Consider the system below:

$$x_e[n] \qquad \tilde{x}_i[n]$$

| | ↑ L | | LPF Gain = L cutoff | | ↓ M | |

$x[n]$ → ↑ L → $x_e[n]$ → LPF, Gain = L, cutoff → $\tilde{x}_i[n]$ → ↓ M → $\tilde{x}_d[n]$

Sampling period: T   T/L   T/L   TM/L

where the cutoff of the LPF is at $\min(\pi/L, \pi/M)$. The Fourier transform of the input signal $x[n]$ is

$X(e^{j\omega})$

$-\pi \qquad -\omega_0 \qquad \omega_0 \qquad \pi$
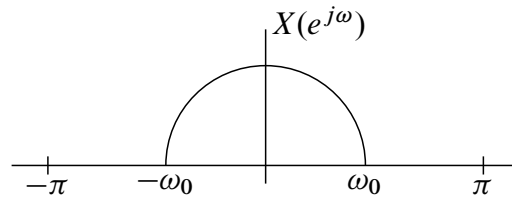
For $M = 5$ and $L = 3$, draw the transforms of the signals at each stage, and specify the maximum value of ω_0 such that $\tilde{X}_d(e^{j\omega}) = aX(e^{jM\omega/L})$ for some a.