EEE2035F EXAM
SIGNALS AND SYSTEMS I

University of Cape Town

Department of Electrical Engineering

June 2016

2 hours

Information

- The exam is closed-book.
- There are *8* questions totaling 75 marks. You must answer all of them.
- Marks are awarded based on method and clarity of presentation. Just writing down the answer is not a good strategy.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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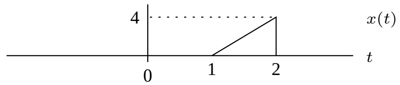
1. Consider the signal below:



Plot the following:

(a) $y_1(t) = x(1-t)$

(b) $y_2(t) = x(t+1/2)\delta(t-1)$

(c) $y_3(t) = x(t/2-1)$

(d) $y_4(t) = \int_{-\infty}^{t} x(\lambda)d\lambda$

(e) $y_5(t) = \frac{d}{dt}x(t)$.

(10 marks)

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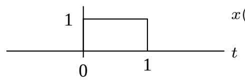
2. Note that the systems in the three parts of this question are different.

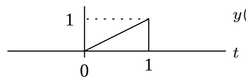
(a) The input $x(t)$ and the output $y(t)$ of a system T_a are related by the equation

$$y(t) = x(t-1) + x(1-t).$$

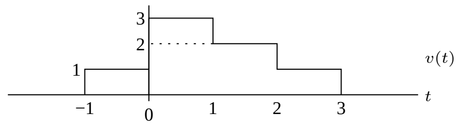
Is the system time invariant?

(b) When the input to a LTI system T_b is $x(t)$ below then the output is $y(t)$:





Find the output when the input is the following:



(c) A LTI system T_c with input $x(t)$ and output $y(t)$ is governed by the input-output relationship

$$y(t) = \frac{1}{3}(x(t+1) + x(t) + x(t-1)).$$

Plot the impulse response of the system.

(10 marks)

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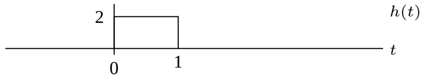
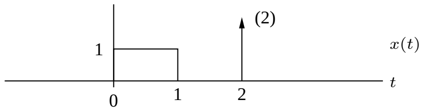
3. A system with impulse response



is driven by an input signal



Find and plot the output $y(t)$.

(10 marks)

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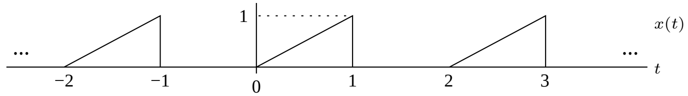
4. The signal



can be written in the form

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\pi t},$$

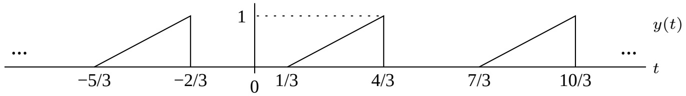
where the coefficients satisfy

$$c_k = \begin{cases} 1/4 & (k = 0) \\ \frac{1}{jk\pi}\left[\frac{1}{k\pi}\sin(k\pi/2)e^{-jk\pi/2} - \frac{1}{2}e^{-jk\pi}\right] & \text{(otherwise).} \end{cases}$$

(a) Show that $|c_2| = \frac{1}{4\pi}$ and $\angle c_2 = \frac{\pi}{2}$. What frequency is associated with this coefficient?

(b) What is the value of the coefficient c_{-2}?

(c) The signal



can be written in the form $y(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\pi t}$. Show that $d_k = c_k e^{-jk\pi/3}$.

(d) What are the values $|d_2|$ and $\angle d_2$?

(10 marks)

5. A LTI system generates the output

$$y(t) = (e^{-2t} - e^{-3t})u(t)$$

in response to the input $x(t) = e^{-2t}u(t)$.

(a) Show that the frequency response of the system is

$$H(\omega) = \frac{1}{j\omega + 3}.$$

(b) Determine the unit impulse response $h(t)$ of the system.

(c) Sketch the magnitude response $|H(\omega)|$ and phase response $\angle H(\omega)$ of the system.

(d) What is the output $y(t)$ of the system when the input is $x(t) = e^{j3t}$?

(10 marks)

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6. A system with input $x(t)$ and output $y(t)$ is described by the differential equation

$$\frac{d^2}{dt^2}y(t) - 5\frac{d}{dt}y(t) + 6y(t) = -\frac{d}{dt}x(t).$$

(a) Show that the frequency response of the system is

$$H(\omega) = \frac{-j\omega}{(j\omega - 3)(j\omega - 2)}.$$

(b) Find the impulse response of the system. You can use the partial fraction expansion below if necessary:

$$\frac{-1}{(s-2)(s-3)} = \frac{1}{(s-2)} - \frac{1}{(s-3)}.$$

(10 marks)

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7. The Fourier transform of signal $x(t)$ is given by

$$X(\omega) = \frac{2}{(j\omega)}(1 - \cos(\omega)).$$

(a) Without calculating $x(t)$ find the Fourier transform $Y(\omega)$ of $y(t) = x(0.5t - 2)$.

(b) Show that the following is a valid Fourier pair:

$$\frac{1}{2}(\delta(t-1) + \delta(t+1)) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \cos(\omega).$$

(c) Find the inverse Fourier transform $x(t)$. You may use the result from (b) if required.

(10 marks)

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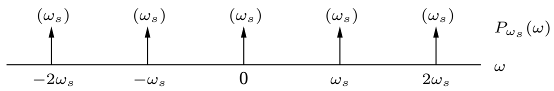
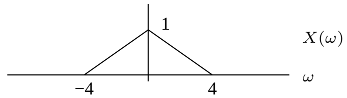
8. The periodic impulse train signal

$$P_{\omega_s}(\omega) = \omega_s \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

is sketched below:



(a) Suppose the signal $x(t)$ has the following spectrum:



Sketch $Y(\omega) = P_{\omega_s}(\omega) * X(\omega)$ for $\omega_s = 10$.

(b) What is the smallest value of ω_s for which the signal $x(t)$ can be recovered from $Y(\omega) = P_{\omega_s}(\omega) * X(\omega)$?

(c) Specify a filter $H(\omega)$ that can be used to reconstruct $x(t)$ from $Y(\omega)$ in part (a).

(5 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t)+bv(t) \leftrightarrow aX(\omega)+bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a>0$
Time reversal	$x(-t) \leftrightarrow X(-\omega)=\overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n}X(\omega) \quad n=1,2,\ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega-\omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n=1,2,\ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Convolution in time domain	$x(t)*v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega)*V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega t}d\omega$	$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$
$1 \quad (-\infty<t<\infty)$	$2\pi\delta(\omega)$
$-0.5+u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega+b} \quad (b>0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0) \quad$ (ω_0 any real number)
$p_\tau(t)$	$\tau\,\mathrm{sinc}\frac{\tau\omega}{2\pi}$
$\tau\,\mathrm{sinc}\frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1-\frac{2\lvert t\rvert}{\tau}\right)p_\tau(t)$	$\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\mathrm{sinc}^2\frac{\tau t}{4\pi}$	$2\pi\left(1-\frac{2\lvert\omega\rvert}{\tau}\right)p_\tau(\omega)$
$\cos(\omega_0 t+\theta)$	$\pi[e^{-j\theta}\delta(\omega+\omega_0)+e^{j\theta}\delta(\omega-\omega_0)]$
$\sin(\omega_0 t+\theta)$	$j\pi[e^{-j\theta}\delta(\omega+\omega_0)-e^{j\theta}\delta(\omega-\omega_0)]$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-k\frac{2\pi}{T})$

with $p_\tau(t)=u(t+\tau/2)-u(t-\tau/2)$ and $\mathrm{sinc}(\lambda)=\sin(\pi\lambda)/(\pi\lambda)$.

Trigonometric identities

$\sin(-\theta)=-\sin(\theta) \qquad \cos(-\theta)=\cos(\theta) \qquad \tan(-\theta)=-\tan(\theta) \qquad \sin^2(\theta)+\cos^2(\theta)=1$

$\sin(2\theta)=2\sin(\theta)\cos(\theta) \qquad \cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)=2\cos^2(\theta)-1=1-2\sin^2(\theta)$

$\sin(\theta_1+\theta_2)=\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1+\theta_2)=\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2)$

$e^{j\theta}=\cos(\theta)+j\sin(\theta)$