EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2015

1. *Here (a) is a shift to the right by 2, (b) is time reversed with a subsequent shift to take the origin to* $t = 2$*, (c) involves compression of the signal by a factor of two around the origin, coupled by an overall scaling of the range by* -2 *(i.e. upside down and bigger), (d) involves compression of the signal by a factor of two around the origin followed by a shift to take the origin to* $t = 1$*, (e) is the indefinite integral (*0 *before* $t = -1$*,* 2 *at* $t = 1$*,* 1.5 *for* $t > 2$*, with linear pieces in between), and (f) is zero everywhere except for delta functions at* $t = -1$*,* $t = 1$ *and* $t = 2$ *with sizes* 1*,* -1.5*, and* 1.5 *respectively.*

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2. (a) *Impulse response* $h(t)$ *is output when input is Dirac delta, or* $i(t) = \delta(t)$*, so* $h(t) = u(t)$*.*

   (b) *To calculate output at* $t = t_0$ *require knowledge of input for* $-\infty < t \leq t_0$*. These values all in past, so causal, and memory is required. Could also note that impulse response* $h(t)$ *is zero for* $t < 0$ *so causal.*

   (c) *Supposing input* $i(t) = i_0(t)$ *gives output* $q_0(t) = \int_{-\infty}^{t} i_0(\phi) d\phi$*. Output for* $i(t) = i_0(t - c)$ *is* $q(t) = \int_{-\infty}^{t} i_0(\phi - c) d\phi = \int_{-\infty}^{t-c} i_0(\psi) d\psi = q_0(t - c)$ *so time invariant.*

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3. *With* $z(t) = h(t) * u(t)$ *the derivative property states that* $\dot{z}(t) = h(t) * u(t) = h(t) * \delta(t) = h(t)$*, so can find* $z(t)$ *by indefinite integration. Then use* $y(t) = h(t) * u(t - 1) = z(t - 1)$ *and plot as required.*

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4. (a) *Exponential decaying to the right, truncated to start at* $t = -1$*.*

   (b) *Impulse response is derivative of step response. Zero for* $t < -1$ *and turns out* to be negative for $t > -1$. Remember delta function at $t = -1$ to take care of discontinuity.

(c) Impulse response not zero for all $t < 0$ so not causal.

(d) The given $g(t)$ is the output when the input is $u(t)$. The input $3u(t+4)$ is just a scaled and shifted version of the previous input, so the output in this case will be $3g(t+4)$.

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5. (a) Just substitute and get values of c_k for $k \geq 0$. The values will in this instance be real, but nonetheless just plot them in the Argand diagram, read off the magnitude and phase, and plot accordingly. Since $x(t)$ is real the coefficients will satisfy $c_k = c_{-k}^*$ (i.e. magnitudes equal but phases will have opposite sign).

(b) Third harmonic is $c_{-3}e^{-j3\pi t} + c_3 e^{j3\pi t}$. By Parseval's theorem the total power will be $|c_{-3}|^2 + |c_3|^2 = 2|c_3|^2$ which can easily be calculated.

(c) Here $y(t) = 2x(t-0.5) = 2\sum_{-\infty}^{\infty} c_k e^{jk\pi(t-0.5)} = \sum_{-\infty}^{\infty} 2c_k e^{-jk\pi 0.5} e^{jk\pi t}$, which is the form of a Fourier series with coefficients $d_k = 2c_k e^{-jk\pi/2}$ and the values can be found.

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6. (a) Take Fourier transforms of both sides, use the derivative property, and manipulate to find $Y(\omega)/X(\omega) = H(\omega)$.

(b) Use $|H(\omega)|^2 = H(\omega)H^*(\omega)$ to find $|H(\omega)| = \omega RC/sqrt1 + (\omega RC)^2$. Observe that $|H(0)| = 0$ and $|H(\omega)| \to 1$ as $\omega \to \pm\infty$ and plot accordingly. This is a first-order highpass filter.

(c) Frequency response can be written as $H(\omega) = a + b/(\frac{1}{RC} + j\omega)$. Solve for unknown coefficients and invert for impulse response $h(t) = \delta(t) - \frac{1}{RC}e^{-\frac{1}{RC}t}u(t)$.

(d) Input is $x(t) = 1/2e^{j5\pi t} + 1/2e^{-j5\pi t}$ so using fundamental properties of the frequency response the output will be $y(t) = \frac{1}{2}H(5\pi)e^{j5\pi t} + \frac{1}{2}H(-5\pi)e^{-j5\pi t}$. Find $H(5\pi)$ and $H(-5\pi)$ (they will be conjugates) and the result can be written as a real cosine again if required.

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7. (a) Use frequency translation on pair $e^{-3t}u(t) \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{1}{3+j\omega}$ and then time shifting to make the right-hand side look like $X(\omega)$. The left-hand side is then the required

$x(t) = 6e^{-3(t+1)}u(t+1)e^{j4(t+1)}$.

(b) *Time scaling to the given transform provides* $e^{-|at|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2a}{a^2+\omega^2}$. *Then apply duality followed by frequency shift to get* $X(\omega) = \frac{\pi}{a}e^{-|a(\omega-W)|}$.

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8. (a) *The required spectrum will contain two half-amplitude replicas of* $G(\omega)$ *centered at* $\omega = \pm 20$ *rad/s.*

(b) *The given system is an ideal lowpass filter that keeps content between* $-20 \leq omega \leq 20$, *so the "outside" halves of each replica in* $G(\omega)$ *will be eliminated. Easiest to see graphically but you should get* $Q(\omega) = 1/2p_{10}(\omega + 15) + 1/2p_{10}(\omega - 15)$.

(c) *Total energy is* $E = 1/(2\pi)\int_{-\infty}^{\infty} |Q(\omega)|^2 d\omega = 2(10/4)/(2\pi)$ *(Joules).*

(d) *Since* $\omega_{max} = 10$ *and must sample with a frequency* $\omega = 2\pi/T > 2\omega_{max}$, *require* $T < \pi/10$ *seconds.*

(e) *Pulse train* $g(t)$ *has transform* $P(\omega) = 2\pi/T_s \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$ *with* $\omega_s = 2\pi/T_s = 30$ *rad/s. Multiplication in time is convolution in frequency, so* $G_s(\omega) = 1/T_s \sum_{k=-\infty}^{\infty} G(\omega - k\omega_s)$ *is the required result and contains scaled replicas of* $G(\omega)$ *at spacings* ω_s.
