EEE2035F EXAM
SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2015
2 hours

Information

- The exam is closed-book.
- There are *8* questions totaling 70 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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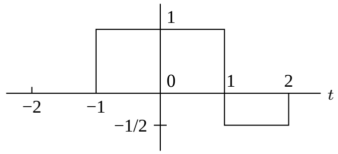
1. Let $g(t)$ be the function



Sketch the following:

(a) $y_1(t) = g(t-2)$

(b) $y_2(t) = g(2-t)$

(c) $y_3(t) = -2g(2t)$

(d) $y_4(t) = g(2t-2)$

(e) $y_5(t) = \int_{-\infty}^{t} g(\tau)d\tau$

(f) The generalised derivative $y_6(t) = \frac{d}{dt}g(t)$.

(10 marks)

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2. The charge $q(t)$ accumulated by an ideal capacitor C is related to the current $i(t)$ passing through it by

$$q(t) = \int_{-\infty}^{t} i(\phi)d\phi.$$

(a) With the input $i(t)$ and output $q(t)$, what is the impulse response of the capacitor?

(b) Is the system causal? Does it have memory? Explain, with reasons.

(c) Show that the system is time invariant.

(5 marks)

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3. Find and plot $y(t) = h(t) * x(t)$ with

$$h(t) = e^{-t}u(t) \qquad \text{and} \qquad x(t) = u(t-1).$$

(5 marks)

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4. A linear time-invariant system has the following *step* response

$$g(t) = 2e^{-4t}u(t+1).$$

(a) Sketch the step response as function of time.

(b) Find the impulse response $h(t)$.

(c) Is this system causal or non-causal? Justify your answer.

(d) What is the output if the input is $x(t) = 3u(t+4)$?

(10 marks)

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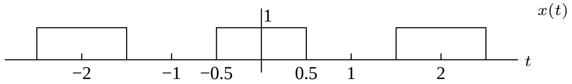
5. The signal



has a Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\pi t},$$

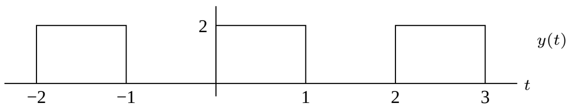
where

$$c_k = \begin{cases} 1/2 & k = 0 \\ \frac{1}{k\pi}\sin(k\pi/2) & \text{otherwise.} \end{cases}$$

(a) Plot the magnitude and phase of the coefficients c_k as a function of k over the range -4 to 4.

(b) How much power is contained in the third harmonic of $x(t)$?

(c) Use the information given to find a Fourier series expansion for the signal $y(t)$ below:



(10 marks)

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6. The input $x(t)$ and output $y(t)$ of a particular RC circuit obeys the differential equation

$$y(t) = RC\left(\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right).$$

(a) Show that the frequency response of the system is

$$H(\omega) = \frac{j\omega RC}{1 + j\omega RC}.$$

(b) Plot $|H(\omega)|$. Does the circuit have a lowpass or a highpass transfer characteristic?

(c) Find the impulse response of the system.

(d) What is the output of the system if the input is $x(t) = \cos(5\pi t)$ and $RC = 1$? This output should be real but you may express your result in terms of complex quantities.

(10 marks)

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7. (a) Find the inverse Fourier transform of the function

$$X(\omega) = \frac{6}{3 + j\omega - j4}e^{j\omega}.$$

(b) Use the valid transform pair

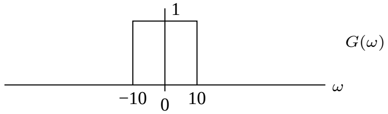
$$e^{-|t|} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \frac{2}{1+\omega^2}$$

to find the Fourier transform of

$$x(t) = \frac{e^{jWt}}{a^2 + t^2}.$$

(10 marks)

8. Consider a signal $g(t)$ with the following spectrum:



(a) Plot the frequency spectrum of $f(t) = g(t)\cos(20t)$.

(b) Suppose $f(t)$ is now filtered using an LTI system with frequency response

$$H(\omega) = \begin{cases} 1 & |\omega| \leq 20 \\ 0 & \text{otherwise.} \end{cases}$$

to produce the output $q(t)$. Plot the frequency spectrum $Q(\omega)$ of $q(t)$.

(c) Calculate the total energy in $q(t)$.

(d) According to Nyquist, what is the largest sampling interval T for which the signal $g(t)$ can be reconstructed from samples $g[n] = g(nT)$?

(e) Sketch the spectrum of $g_s(t) = g(t)p(t)$, where

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - T_s k)$$

and $T_s = \pi/15$ seconds.

(10 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n}X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \operatorname{sinc} \frac{\tau\omega}{2\pi}$
$\tau \operatorname{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\operatorname{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega - k\frac{2\pi}{T})$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta) \qquad \sin^2(\theta) + \cos^2(\theta) = 1$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$