EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2013

1. (a) *Flip around origin and scale range by 2*

   (b) *Flip around origin and shift origin to* $t = 2$

   (c) *Ordinary derivative is 1 for* $1 \leq t \leq 2$ *and zero elsewhere, but also need* $-\delta(t-2)$ *to account for discontinuity*

   (d) *Parabolic and concave upwards from* $t = 1$ *to* $t = 2$*, attaining a value of* $1/2$ *at* $t = 2$*. Constant value of* $1/2$ *for* $t > 2$*, and zero for* $t < 1$*.*

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2. (a) $y_1(t) = x_1(t)u(t) = \delta(t-1)u(t) = \delta(t-1)$.

   (b) $y_2(t) = x_2(t)u(t) = \delta(t+1)u(t) = 0$.

   (c) *In above have* $x_1(t) = x_2(t-2)$ *but* $y_1(t) \neq y_2(t-2)$*, so the system not time invariant.*

   (d) *Suppose* $y_3(t)$ *is the response to* $x_3(t)$*: then* $y_3(t) = x_3(t)u(t)$*. For any* a *the response to* $x_4(t) = ax_3(t)$ *is* $y_4(t) = x_4(t)u(t) = ax_3(t)u(t) = ay_3(t)$*, so the system is homogeneous.*

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3. *Here* $h(t) = p_2(t)$ *is a pulse with height 1 for* $-1 \leq t \leq 1$ *and zero elsewhere. Using graphical or other methods the required output can be shown to be*

$$y(t) = p_2(t) * p_1(t - 1/2) = \begin{cases} t+1 & -1 \leq t \leq 0 \\ 1 & 0 < t < 1 \\ 2-t & 1 \leq t \leq 2 \\ 0 & \textit{otherwise}. \end{cases}$$

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4. *Fundamental period* $T = 6$ *seconds so fundamental frequency is* $\omega_0 = \frac{2\pi}{6} = \pi/3$ *radians per second.*

   (a) *Coefficient for the* $k = 0$ *term is the DC value:*
$$c_0 = \frac{1}{3}.$$

   (b) *In general* $x(t) = 2z(t) - z(t-3)$*, so substituting and simplifying gives*
$$x(t) = 2\sum_{k=-\infty}^{\infty} d_k e^{jk\frac{\pi}{3}t} - \sum_{k=-\infty}^{\infty} d_k e^{jk\frac{\pi}{3}(t-3)} = \sum_{k=-\infty}^{\infty} \left(2d_k - e^{jk\pi} d_k\right) e^{jk\frac{\pi}{3}t}.$$
   This is in the form of a Fourier series
$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\frac{\pi}{3}t}$$
   with $c_k = (2 - e^{jk\pi})d_k$*. Thus*
$$c_3 = (2 - e^{j3\pi})d_3 = (2 - e^{j3\pi})0 = 0.$$

   (c) *Because* $x(t)$ *is real we have* $c_{-3} = c_3^* = 0$*.*

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5. (a) *Time reversal on pair* $e^{-bt}u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad 1/(j\omega + b)$ *gives required transform*
$$X(\omega) = \frac{1}{-j\omega + 2}.$$

   (b) *Starting with the pair for* $3e^{-bt}u(t)$ *and applying time shift followed by multiplication by power of* t *gives required signal is* $x(t) = 3te^{-3(t+2)}u(t+2)$*.*

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6. (a) *Take Fourier transform of each side and manipulate for required expression.*

   (b) *Output is*
$$Y(\omega) = H(\omega)X(\omega) = \frac{2}{(j\omega + 3)^2(j\omega + 1)}.$$

   (c) *Using partial fractions inverse transform gives the impulse response*
$$h(t) = -e^{-3t}u(t) + e^{-t}u(t).$$

7.

8. *(a)* *Using tables with* $\tau = 2$ *gives* $X(\omega) = 2\pi(1 - |\omega|)p_2(\omega)$ *(triangular pulse centred on zero, total width 2, and height* 2π.

   (b) *Highest frequency present is 1 radian/second so Nyquist requires that* $\omega_s = \frac{2\pi}{T} > 2(1)$, *or* $T < \pi$ *seconds.*

   (c) *Here* $\omega_s = \frac{2\pi}{T_s} = 3$ *radians/second, so the spectrum* $X_s(\omega)$ *consists of replicas from part (a) occurring at integer multiples of* $\omega_s = 3$, *each with height 3.*