EEE2035F EXAM SIGNALS AND SYSTEMS I

University of Cape Town Department of Electrical Engineering

June 2013 2 hours

Information

- The exam is closed-book.
- There are 8 questions totaling 75 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

1. Consider x(t) below



Sketch the following:

- (a) $x_1(t) = 2x(-t)$
- (b) $x_2(t) = x(-t+2)$
- (c) $x_3(t) = \frac{d}{dt}x(t)$
- (d) $x_4(t) = \int_{-\infty}^t x(\tau) d\tau$.

(10 marks)

- 2. Consider a continuous-time system which has input signal x(t) and output y(t) = x(t)u(t), where u(t) is the unit step.
 - (a) Find the output when the input is $x_1(t) = \delta(t-1)$.
 - (b) Find the output when the input is $x_2(t) = \delta(t+1)$.
 - (c) Is this system time invariant? Justify your answer.
 - (d) Is this system homogeneous? Justify your answer.

(10 marks)

3. An LTI system has an impulse response h(t) = u(t+1) - u(t-1). Sketch h(t), and determine and sketch the response of the system to the input below:



(10 marks)

4. Consider the periodic signal x(t) shown below:



Specify the fundamental frequency ω_0 , and in the series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

find the Fourier series coefficients c_k for

- (a) k = 0
- (b) k = 3
- (c) k = -3.

You may wish to make use of the fact that the signal z(t) below



has a Fourier series expansion of the form

$$z(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\frac{\pi}{3}t}$$

with

$$d_k = \begin{cases} \frac{1}{3} & k = 0\\ \frac{1}{k\pi} \sin(\frac{k\pi}{3}) & \text{otherwise.} \end{cases}$$

(10 marks)

5. (a) Find the Fourier transform of the signal

$$x(t) = e^{2t}u(-t).$$

(b) A signal x(t) can be expressed in the frequency domain as

$$X(\omega) = j \frac{d}{d\omega} \left\{ \frac{e^{j2\omega}}{1 + j\omega/3} \right\}.$$

Use the properties of the continuous-time Fourier transform to find the time domain signal x(t).

(10 marks)

6. The input and output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2x(t).$$

(a) Show that the frequency response of the system can be written as

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{(j\omega+3)(j\omega+1)}$$

- (b) What is the Fourier transform of the output if the input is $x(t) = e^{-3t}u(t)$?
- (c) Find the impulse response of the system.

(10 marks)

7. The impulse response of an LTI system is given by

$$h(t) = \frac{1}{4}\delta(t+1) + \delta(t) + \frac{1}{4}\delta(t-1).$$

- (a) Is the system causal? Why?
- (b) Find and sketch the magnitude $|H(\omega)|$ and phase $\angle H(\omega)$ of the frequency response of the LTI system over the frequency interval $-2\pi \le \omega \le 2\pi$.
- (c) Suppose the given system is connected via a cascade (series) structure to another system with impulse response $g(t) = p_1(t)$. Determine the overall impulse response of the interconnection of LTI systems.

(10 marks)

8. Consider the signal

$$x(t) = \operatorname{sinc}^2(t/2\pi).$$

- (a) Find and plot $X(\omega)$.
- (b) According to Nyquist, what is the largest sampling interval T for which the signal can be reconstructed from samples?
- (c) Sketch the spectrum of $x_s(t) = x(t)p(t)$, where

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

and $T_s = \frac{2}{3}\pi$.

(5 marks)

INFORMATION SHEET

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega) e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) n = 1, 2, \dots$
Frequency shift	$x(t)e^{j\omega_0t} \leftrightarrow X(\omega-\omega_0) \omega_0 \text{ real}$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) n = 1, 2, \dots$
Integration	$\int_{-\infty}^{t} x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^{2}(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^{2} d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Fourier transform properties

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$	
1 $(-\infty < t < \infty)$	$2\pi\delta(\omega)$	
-0.5 + u(t)	$\frac{1}{j\omega}$	
u(t)	$\pi\delta(\omega) + \frac{1}{j\omega}$	
$\delta(t)$	1	
$\delta(t-c)$	$e^{-j\omega c}$ (c any real number)	
$e^{-bt}u(t)$	$\frac{1}{j\omega+b}$ $(b>0)$	
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$ (ω_0 any real number)	
$p_{ au}(t)$	$\tau \operatorname{sinc} \frac{\tau \omega}{2\pi}$	
$\tau \operatorname{sinc} \frac{\tau t}{2\pi}$	$2\pi p_{ au}(\omega)$	
$\left(1-rac{2 t }{ au} ight)p_{ au}(t)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$	
$\frac{\tau}{2}$ sinc ² $\frac{\tau t}{4\pi}$	$2\pi \left(1 - rac{2 \omega }{ au} ight) p_{ au}(\omega)$	
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega+\omega_0)+e^{j\theta}\delta(\omega-\omega_0)]$	
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega+\omega_0)-e^{j\theta}\delta(\omega-\omega_0)]$	
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-k\frac{2\pi}{T})$	
with $p_{\tau}(t) = u(t + \tau/2) - u(t - \tau/2)$ and $\operatorname{sinc}(\lambda) = \frac{\sin(\pi \lambda)}{(\pi \lambda)}$.		

Trigonometric identities

 $\begin{aligned} \sin(-\theta) &= -\sin(\theta) & \cos(-\theta) = \cos(\theta) & \tan(-\theta) = -\tan(\theta) & \sin^2(\theta) + \cos^2(\theta) = 1\\ \sin(2\theta) &= 2\sin(\theta)\cos(\theta) & \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)\\ \sin(\theta_1 + \theta_2) &= \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) & \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)\\ e^{j\theta} &= \cos(\theta) + j\sin(\theta) \end{aligned}$