# EEE2035F EXAM
# SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2013
2 hours

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**Information**

- The exam is closed-book.
- There are *8* questions totaling 75 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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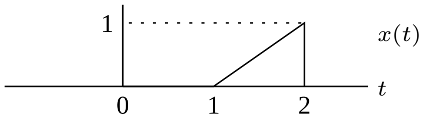
1. Consider $x(t)$ below




Sketch the following:

(a) $x_1(t) = 2x(-t)$

(b) $x_2(t) = x(-t+2)$

(c) $x_3(t) = \frac{d}{dt}x(t)$

(d) $x_4(t) = \int_{-\infty}^{t} x(\tau)d\tau$.

(10 marks)

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2. Consider a continuous-time system which has input signal $x(t)$ and output $y(t) = x(t)u(t)$, where $u(t)$ is the unit step.

(a) Find the output when the input is $x_1(t) = \delta(t-1)$.

(b) Find the output when the input is $x_2(t) = \delta(t+1)$.

(c) Is this system time invariant? Justify your answer.

(d) Is this system homogeneous? Justify your answer.

(10 marks)

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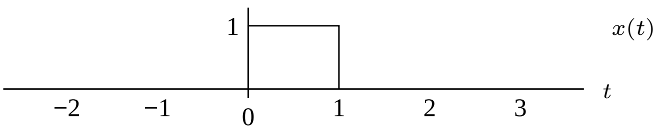
3. An LTI system has an impulse response $h(t) = u(t+1) - u(t-1)$. Sketch $h(t)$, and determine and sketch the response of the system to the input below:




(10 marks)

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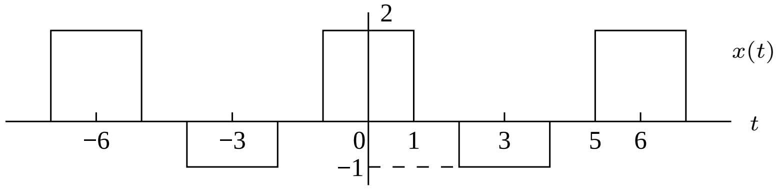
4. Consider the periodic signal $x(t)$ shown below:




Specify the fundamental frequency $\omega_0$, and in the series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

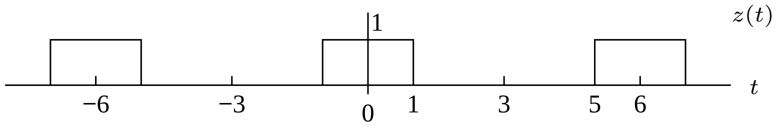
find the Fourier series coefficients $c_k$ for

(a) $k = 0$

(b) $k = 3$

(c) $k = -3$.

You may wish to make use of the fact that the signal $z(t)$ below




has a Fourier series expansion of the form

$$z(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\frac{\pi}{3}t}$$

with

$$d_k = \begin{cases} \frac{1}{3} & k = 0 \\ \frac{1}{k\pi}\sin(\frac{k\pi}{3}) & \text{otherwise.} \end{cases}$$

(10 marks)

5. (a) Find the Fourier transform of the signal

$$x(t) = e^{2t}u(-t).$$

(b) A signal $x(t)$ can be expressed in the frequency domain as

$$X(\omega) = j\frac{d}{d\omega}\left\{\frac{e^{j2\omega}}{1 + j\omega/3}\right\}.$$

Use the properties of the continuous-time Fourier transform to find the time domain signal $x(t)$.

(10 marks)

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6. The input and output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2x(t).$$

(a) Show that the frequency response of the system can be written as

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{(j\omega + 3)(j\omega + 1)}.$$

(b) What is the Fourier transform of the output if the input is $x(t) = e^{-3t}u(t)$?

(c) Find the impulse response of the system.

(10 marks)

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7. The impulse response of an LTI system is given by

$$h(t) = \frac{1}{4}\delta(t + 1) + \delta(t) + \frac{1}{4}\delta(t - 1).$$

(a) Is the system causal? Why?

(b) Find and sketch the magnitude $|H(\omega)|$ and phase $\angle H(\omega)$ of the frequency response of the LTI system over the frequency interval $-2\pi \leq \omega \leq 2\pi$.

(c) Suppose the given system is connected via a cascade (series) structure to another system with impulse response $g(t) = p_1(t)$. Determine the overall impulse response of the interconnection of LTI systems.

(10 marks)

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8. Consider the signal

$$x(t) = \text{sinc}^2(t/2\pi).$$

(a) Find and plot $X(\omega)$.

(b) According to Nyquist, what is the largest sampling interval $T$ for which the signal can be reconstructed from samples?

(c) Sketch the spectrum of $x_s(t) = x(t)p(t)$, where

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

and $T_s = \frac{2}{3}\pi$.

(5 marks)

# INFORMATION SHEET

## Fourier transform properties

| Property | Transform Pair/Property |
|---|---|
| Linearity | $ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$ |
| Time shift | $x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$ |
| Time scaling | $x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$ |
| Time reversal | $x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$ |
| Multiplication by power of $t$ | $t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$ |
| Frequency shift | $x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real |
| Multiplication by $\cos(\omega_0 t)$ | $x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$ |
| Differentiation in time domain | $\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$ |
| Integration | $\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$ |
| Convolution in time domain | $x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$ |
| Multiplication in time domain | $x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$ |
| Parseval's theorem | $\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$ |
| Parseval's theorem (special case) | $\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \|X(\omega)\|^2 d\omega$ |
| Duality | $X(t) \leftrightarrow 2\pi x(-\omega)$ |

## Common Fourier Transform Pairs

| $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$ | $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ |
|---|---|
| $1 \quad (-\infty < t < \infty)$ | $2\pi\delta(\omega)$ |
| $-0.5 + u(t)$ | $\frac{1}{j\omega}$ |
| $u(t)$ | $\pi\delta(\omega) + \frac{1}{j\omega}$ |
| $\delta(t)$ | $1$ |
| $\delta(t-c)$ | $e^{-j\omega c} \quad$ ($c$ any real number) |
| $e^{-bt}u(t)$ | $\frac{1}{j\omega + b} \quad (b > 0)$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0) \quad$ ($\omega_0$ any real number) |
| $p_\tau(t)$ | $\tau \mathrm{sinc}\,\frac{\tau\omega}{2\pi}$ |
| $\tau \mathrm{sinc}\,\frac{\tau t}{2\pi}$ | $2\pi p_\tau(\omega)$ |
| $\left(1 - \frac{2\|t\|}{\tau}\right) p_\tau(t)$ | $\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$ |
| $\frac{\tau}{2}\mathrm{sinc}^2\,\frac{\tau t}{4\pi}$ | $2\pi\left(1 - \frac{2\|\omega\|}{\tau}\right) p_\tau(\omega)$ |
| $\cos(\omega_0 t + \theta)$ | $\pi[e^{-j\theta}\delta(\omega+\omega_0) + e^{j\theta}\delta(\omega-\omega_0)]$ |
| $\sin(\omega_0 t + \theta)$ | $j\pi[e^{-j\theta}\delta(\omega+\omega_0) - e^{j\theta}\delta(\omega-\omega_0)]$ |
| $\sum_{n=-\infty}^{\infty}\delta(t - nT)$ | $\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - k\frac{2\pi}{T})$ |

with $p_\tau(t) = u(t+\tau/2) - u(t-\tau/2)$ and $\mathrm{sinc}(\lambda) = \sin(\pi\lambda)/(\pi\lambda)$.

## Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta) \qquad \sin^2(\theta) + \cos^2(\theta) = 1$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$