

EEE2035F EXAM SIGNALS AND SYSTEMS I

HINTS: June 2012

1. (a) *The slope for the ordinary derivative is $1/6$ over the range 6 to 12, but the generalised derivative must include an impulse of size -1 at 12 to take into account the discontinuity.*
(b) *Result at t is the area under g to the left of t at each point. Note that $x(t) = 3$ for $t \rightarrow \infty$, and $x(t)$ is quadratic and concave upwards between 6 and 12.*
(c) *Since $y(t) = g(4 - 2t) = g(-2(t - 2)) = g_2(t - 2)$ one could plot $g_2(t) = g(-2t)$ (flip around origin and compression by a factor 2), and then shift the result to the right (delay) by 2 to get $y(t)$.*
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2. *First sketch the signals: $x(t)$ is a rectangular pulse of height 2 and total width 1 centred on $t = 3/2$, and $h(t)$ is a rectangular pulse of height -1 and total width 2 centred on $t = 1$. These two signals must be convolved to get the solution, which can be done graphically or (more easily) using the derivative property. The solution is*

$$y(t) = \begin{cases} -2t + 2 & 1 \leq t \leq 2 \\ -2 & 2 \leq t \leq 3 \\ 2t - 8 & 3 \leq t \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

3. (a) *Considering for example $t = 1$ we have $y(1) = x(2)$, so the output depends on a future input and the system is not causal.*
(b) *For input $x_1(t) = u(t)$ the output is $y_1(t) = x_1(2t) = u(2t) = u(t)$ (since compressing the unit step by any positive factor does not change it).*

- (c) For input $x_2(t) = u(t - 1)$ the output is
 $y_2(t) = x_2(2t) = u(2t - 1) = u(2(t - 1/2)) = u(t - 1/2)$ (for the same reason as before).
- (d) For the inputs we have $x_2(t) = x_1(t - 1)$ but $y_2(t) \neq y_1(t - 1)$, so not time invariant.
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4. (a) Start with the transform pair for $e^{-2t}u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{j\omega + 2}$ and apply the multiplication by t property to get a Fourier pair that solves the problem.
- (b) Find $X(\omega)$ and $H(\omega)$ using standard tables and multiply to get the convolution result in the frequency domain: $Y(\omega) = H(\omega)X(\omega)$. The inverse Fourier transform applied to a partial fraction expansion gives the time domain result
 $y(t) = e^{-2t}u(t) - e^{-4t}u(t)$.
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5. (a) Directly evaluate the integral

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-1/2}^{1/2} e^{-j\omega t}dt.$$

- (b) Let $v(t) = \dot{z}(t)$ and observe that $v(t) = x(t) - \delta(t - 1/2)$. Find $V(\omega)$ and use the integration property to get the required transform for $Z(\omega)$ (noting that $V(0) = 0$).
- (c) Substitute expressions into $Y(\omega) = Z(\omega)X(\omega)$ to get required result.
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6. (a) Using tables (or otherwise) we have

$$X(\omega) = \pi\delta(\omega + 25) + \pi\delta(\omega - 25) + \pi\delta(\omega + 60) + \pi\delta(\omega - 60).$$

- (b) Writing the signal as

$$x(t) = \frac{1}{2}e^{j25t} + \frac{1}{2}e^{-j25t} + \frac{1}{2}e^{j60t} + \frac{1}{2}e^{-j60t}$$

we can take $\omega_0 = 5$ radians/second with nonzero coefficients

$$c_5 = c_{-5} = c_{12} = c_{-12} = 1/2.$$

- (c) The power is $P = \sum_{k=-\infty}^{\infty} |c_k|^2 = 4(1/2)^2 = 1$ Watt.
- (d) Input $e^{j\omega_0 t}$ produces output $H(\omega_0)e^{j\omega_0 t}$. Applying to all terms in (b) and simplifying gives $y(t) = \cos(25t - \pi/4) + 2\cos(60t)$.
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7. (a) From the sifting property $w(t) = \delta(t-3)\frac{4-j3^2}{2t} = (\frac{2}{3} - \frac{3}{2}j)\delta(t-3)$.

(b) The only difficult part here is finding the transform of the first term $\cos(5t)u(t)$. One approach is to let $x_1(t) = \cos(5t)$ and $x_2(t) = u(t)$: multiplication in time then corresponds to convolution in frequency, and the final result can be found to be

$$X(\omega) = \frac{1}{2} \left(\pi\delta(\omega+5) + \frac{1}{j(\omega+5)} + \pi\delta(\omega-5) + \frac{1}{j(\omega-5)} \right) + \frac{1}{j\omega+2}.$$

(c) Apply differentiation in time property to the pair $e^{-t}u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{j\omega+1}$ and manipulate for final result $y(t) = -5j[\delta(t) - e^{-t}u(t)]$.

8. (a) Using tables with $\tau = 2$ gives $X(\omega) = 2\pi(1 - |\omega|)p_2(\omega)$ (triangular pulse centred on zero, total width 2, and height 2π).

(b) Highest frequency present is 1 radian/second so Nyquist requires that $\omega_s = \frac{2\pi}{T} > 2(1)$, or $T < \pi$ seconds.

(c) Here $\omega_s = \frac{2\pi}{T_s} = 3$ radians/second, so the spectrum $X_s(\omega)$ consists of replicas from part (a) occurring at integer multiples of $\omega_s = 3$, each with height 3.
