EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2012

1. (a) *The slope for the ordinary derivative is $1/6$ over the range 6 to 12, but the generalised derivative must include an impulse of size -1 at 12 to take into account the discontinuity.*

   (b) *Result at t is the area under g to the left of t at each point. Note that $x(t) = 3$ for $t \to \infty$, and $x(t)$ is quadratic and concave upwards between 6 and 12.*

   (c) *Since $y(t) = g(4 - 2t) = g(-2(t - 2)) = g_2(t - 2)$ one could plot $g_2(t) = g(-2t)$ (flip around origin and compression by a factor 2), and then shift the result to the right (delay) by 2 to get $y(t)$.*

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2. *First sketch the signals: $x(t)$ is a rectangular pulse of height 2 and total width 1 centred on $t = 3/2$, and $h(t)$ is a rectangular pulse of height -1 and total width 2 centred on $t = 1$. These two signals must be convolved to get the solution, which can be done graphically or (more easily) using the derivative property. The solution is*

$$y(t) = \begin{cases} -2t + 2 & 1 \leq t \leq 2 \\ -2 & 2 \leq t \leq 3 \\ 2t - 8 & 3 \leq t \leq 4 \\ 0 & \textit{otherwise}. \end{cases}$$

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3. (a) *Considering for example $t = 1$ we have $y(1) = x(2)$, so the output depends on a future input and the system is not causal.*

   (b) *For input $x_1(t) = u(t)$ the output is $y_1(t) = x_1(2t) = u(2t) = u(t)$ (since compressing the unit step by any positive factor does not change it).*

(c) *For input* $x_2(t) = u(t-1)$ *the output is*
$y_2(t) = x_2(2t) = u(2t-1) = u(2(t-1/2)) = u(t-1/2)$ *(for the same reason as before).*

(d) *For the inputs we have* $x_2(t) = x_1(t-1)$ *but* $y_2(t) \neq y_1(t-1)$, *so not time invariant.*

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4. (a) *Start with the transform pair for* $e^{-2t}u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{j\omega+2}$ *and apply the multiplication by* t *property to get a Fourier pair that solves the problem.*

(b) *Find* $X(\omega)$ *and* $H(\omega)$ *using standard tables and multiply to get the convolution result in the frequency domain:* $Y(\omega) = H(\omega)X(\omega)$. *The inverse Fourier transform applied to a partial fraction expansion gives the time domain result*
$y(t) = e^{-2t}u(t) - e^{-4t}u(t)$.

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5. (a) *Directly evaluate the integral*

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-1/2}^{1/2} e^{-j\omega t}dt.$$

(b) *Let* $v(t) = \dot{z}(t)$ *and observe that* $v(t) = x(t) - \delta(t-1/2)$. *Find* $V(\omega)$ *and use the integration property to get the required transform for* $Z(\omega)$ *(noting that* $V(0) = 0$*).*

(c) *Substitute expressions into* $Y(\omega) = Z(\omega)X(\omega)$ *to get required result.*

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6. (a) *Using tables (or otherwise) we have*

$$X(\omega) = \pi\delta(\omega+25) + \pi\delta(\omega-25) + \pi\delta(\omega+60) + \pi\delta(\omega-60).$$

(b) *Writing the signal as*

$$x(t) = \frac{1}{2}e^{j25t} + \frac{1}{2}e^{-j25t} + \frac{1}{2}e^{j60t} + \frac{1}{2}e^{-j60t}$$

we can take $\omega_0 = 5$ *radians/second with nonzero coefficients*
$c_5 = c_{-5} = c_{12} = c_{-12} = 1/2$.

(c) *The power is* $P = \sum_{k=-\infty}^{\infty} |c_k|^2 = 4(1/2)^2 = 1$ *Watt.*

(d) *Input* $e^{j\omega_0 t}$ *produces output* $H(\omega_0)e^{j\omega_0 t}$. *Applying to all terms in (b) and simplifying gives* $y(t) = \cos(25t - \pi/4) + 2\cos(60t)$.

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7. *(a)* *From the sifting property* $w(t) = \delta(t-3)\frac{4-j3^2}{2t} = (\frac{2}{3} - \frac{3}{2}j)\delta(t-3)$.

   (b) *The only difficult part here is finding the transform of the first term* $\cos(5t)u(t)$. *One approach is to let* $x_1(t) = \cos(5t)$ *and* $x_2(t) = u(t)$*: multiplication in time then corresponds to convolution in frequency, and the final result can be found to be*

$$X(\omega) = \frac{1}{2}\left(\pi\delta(\omega+5) + \frac{1}{j(\omega+5)} + \pi\delta(\omega-5) + \frac{1}{j(\omega-5)}\right) + \frac{1}{j\omega+2}.$$

   (c) *Apply differentiation in time property to the pair* $e^{-t}u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{j\omega+1}$ *and manipulate for final result* $y(t) = -5j[\delta(t) - e^{-t}u(t)]$.

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8. *(a)* *Using tables with* $\tau = 2$ *gives* $X(\omega) = 2\pi(1-|\omega|)p_2(\omega)$ *(triangular pulse centred on zero, total width 2, and height* 2π.

   (b) *Highest frequency present is 1 radian/second so Nyquist requires that* $\omega_s = \frac{2\pi}{T} > 2(1)$, *or* $T < \pi$ *seconds.*

   (c) *Here* $\omega_s = \frac{2\pi}{T_s} = 3$ *radians/second, so the spectrum* $X_s(\omega)$ *consists of replicas from part (a) occurring at integer multiples of* $\omega_s = 3$, *each with height 3.*
