EEE2035F EXAM
SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2012

2 hours

Information

- The exam is closed-book.
- There are *8* questions totalling 75 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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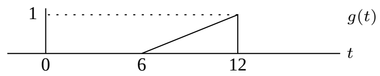
1. Consider the signal $g(t)$ below:



   (a) Sketch the generalised derivative of $g(t)$.

   (b) Sketch $x(t) = \int_{-\infty}^{t} g(\tau)d\tau$.

   (c) Sketch the signal $y(t) = g(4 - 2t)$. Show all the steps.

(10 marks)

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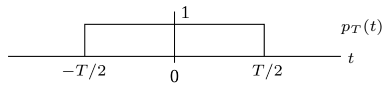
2. Find the convolution of the two signals

$$x(t) = 2p_1(t - 3/2) \qquad \text{and} \qquad h(t) = -p_2(t-1),$$

   where $p_T(t)$ is defined as follows:



(10 marks)

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3. A system is defined by the relationship

$$y(t) = x(2t),$$

   where $x(t)$ is the input and $y(t)$ is the output.

   (a) Is the system causal?

   (b) Find and plot the output when the input is $x_1(t) = u(t)$.

   (c) Find and plot the output when the input is $x_2(t) = u(t-1)$.

   (d) Is the system linear and time invariant?

(10 marks)

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4. (a) Find the Fourier transform of $g(t) = te^{-2t}u(t)$.

(b) Suppose

$$x(t) = 2e^{-2t}u(t) \qquad \text{and} \qquad h(t) = e^{-4t}u(t).$$

Find the Fourier transform of the above signals and use your result to calculate a time-domain expression for $y(t) = x(t) * h(t)$.

(10 marks)

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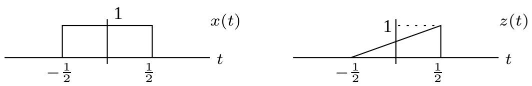
5. Consider two signals $x(t)$ and $z(t)$ shown below:



(a) From the direct definition of the Fourier transform as an integral equation, show that the transform of the signal $x(t)$ is

$$X(\omega) = \frac{2\sin(\omega/2)}{\omega}.$$

(b) Show that the Fourier transform of $z(t)$ is

$$Z(\omega) = \frac{1}{j\omega}\left(\frac{2\sin(\omega/2)}{\omega} - e^{-j\omega/2}\right).$$

(Hint: it is quite easy to find the Fourier transform of the derivative of $z(t)$.)

(c) Suppose a system with impulse response $z(t)$ is driven by an input $x(t)$. Find an expression for the output $Y(\omega)$ in the frequency domain.

(10 marks)

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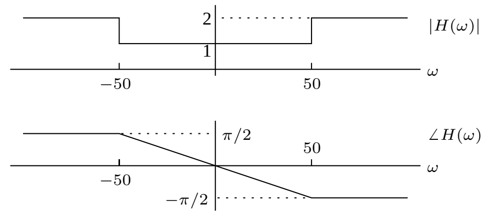
6. The signal $x(t) = \cos(25t) + \cos(60t)$ is provided as input to a linear time-invariant system with the frequency response shown below:



(a) Sketch the magnitude spectrum $|X(\omega)|$ of the signal $x(t)$.

(b) The signal $x(t)$ is periodic, and hence has a Fourier series representation of the form

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}.$$

Show that $\omega_0 = 5$ radians/second and $c_5 = c_{-5} = c_{12} = c_{-12} = \frac{1}{2}$ with all other coefficients zero determines the signal correctly.

(c) Use Parseval's theorem to find the power of the signal $x(t)$.

(d) Determine the output signal $y(t)$.

(10 marks)

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7. (a) Simplify the following expression: $w(t) = \delta(t-3)\frac{4-jt^2}{2t}$.

(b) Find the Fourier transform of $x(t) = (\cos(5t) + e^{-2t})u(t)$.

(c) Find the inverse Fourier transform of $Y(\omega) = \frac{5\omega}{j\omega+1}$.

(10 marks)

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8. Consider the signal

$$x(t) = \text{sinc}^2(t/2\pi).$$

(a) Find and plot $X(\omega)$.

(b) According to Nyquist, what is the largest sampling interval T for which the signal can be reconstructed from samples?

(c) Sketch the spectrum of $x_s(t) = x(t)p(t)$, where

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

and $T_s = \frac{2}{3}\pi$.

(5 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\tau \text{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$