# EEE2035F EXAM
# SIGNALS AND SYSTEMS I

# University of Cape Town
# Department of Electrical Engineering

## June 2012
## 2 hours

## Information

- The exam is closed-book.

- There are *8* questions totalling 75 marks. You must answer all of them.

- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
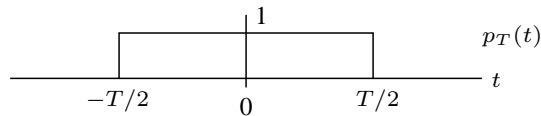
- You have 2 hours.

1. Consider the signal $g(t)$ below:
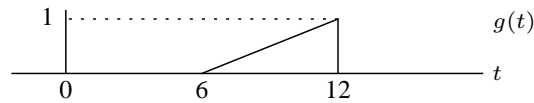


(a) Sketch the generalised derivative of $g(t)$.

(b) Sketch $x(t) = \int_{-\infty}^{t} g(\tau)d\tau$.

(c) Sketch the signal $y(t) = g(4 - 2t)$. Show all the steps.

(10 marks)

2. Find the convolution of the two signals

$$x(t) = 2p_1(t - 3/2) \quad \text{and} \quad h(t) = -p_2(t - 1),$$

where $p_T(t)$ is defined as follows:



(10 marks)

3. A system is defined by the relationship

$$y(t) = x(2t),$$

where $x(t)$ is the input and $y(t)$ is the output.

(a) Is the system causal?

(b) Find and plot the output when the input is $x_1(t) = u(t)$.

(c) Find and plot the output when the input is $x_2(t) = u(t - 1)$.

(d) Is the system linear and time invariant?

(10 marks)

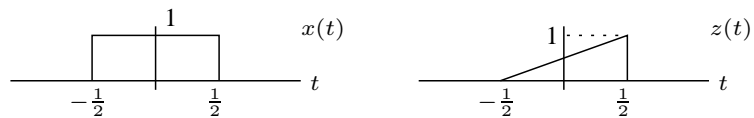4. (a) Find the Fourier transform of $g(t) = te^{-2t}u(t)$.

(b) Suppose

$$x(t) = 2e^{-2t}u(t) \qquad \text{and} \qquad h(t) = e^{-4t}u(t).$$

Find the Fourier transform of the above signals and use your result to calculate a time-domain expression for $y(t) = x(t) * h(t)$.

(10 marks)

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5. Consider two signals $x(t)$ and $z(t)$ shown below:



(a) From the direct definition of the Fourier transform as an integral equation, show that the transform of the signal $x(t)$ is

$$X(\omega) = \frac{2\sin(\omega/2)}{\omega}.$$

(b) Show that the Fourier transform of $z(t)$ is

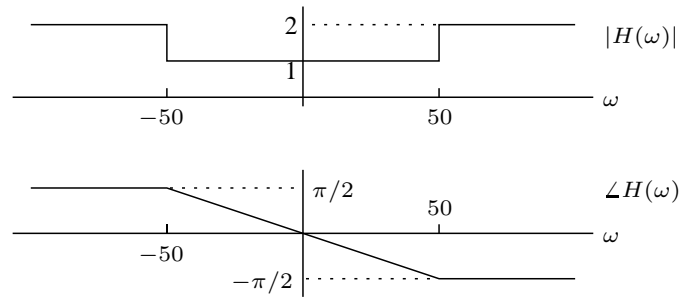$$Z(\omega) = \frac{1}{j\omega}\left(\frac{2\sin(\omega/2)}{\omega} - e^{-j\omega/2}\right).$$

(Hint: it is quite easy to find the Fourier transform of the derivative of $z(t)$.)

(c) Suppose a system with impulse response $z(t)$ is driven by an input $x(t)$. Find an expression for the output $Y(\omega)$ in the frequency domain.

(10 marks)

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6. The signal $x(t) = \cos(25t) + \cos(60t)$ is provided as input to a linear time-invariant system with the frequency response shown below:



(a) Sketch the magnitude spectrum $|X(\omega)|$ of the signal $x(t)$.

(b) The signal $x(t)$ is periodic, and hence has a Fourier series representation of the form

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}.$$

Show that $\omega_0 = 5$ radians/second and $c_5 = c_{-5} = c_{12} = c_{-12} = \frac{1}{2}$ with all other coefficients zero determines the signal correctly.

(c) Use Parseval's theorem to find the power of the signal $x(t)$.

(d) Determine the output signal $y(t)$.

(10 marks)

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7. (a) Simplify the following expression: $w(t) = \delta(t-3)\frac{4-jt^2}{2t}$.

(b) Find the Fourier transform of $x(t) = (\cos(5t) + e^{-2t})u(t)$.

(c) Find the inverse Fourier transform of $Y(\omega) = \frac{5\omega}{j\omega+1}$.

(10 marks)

8. Consider the signal

$$x(t) = \text{sinc}^2(t/2\pi).$$

(a) Find and plot $X(\omega)$.

(b) According to Nyquist, what is the largest sampling interval $T$ for which the signal can be reconstructed from samples?

(c) Sketch the spectrum of $x_s(t) = x(t)p(t)$, where

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

and $T_s = \frac{2}{3}\pi$.

(5 marks)

# INFORMATION SHEET

## Fourier transform properties

| Property | Transform Pair/Property |
|---|---|
| Linearity | $ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$ |
| Time shift | $x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$ |
| Time scaling | $x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a})$   $a > 0$ |
| Time reversal | $x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$ |
| Multiplication by power of $t$ | $t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega)$   $n = 1, 2, \ldots$ |
| Frequency shift | $x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$   $\omega_0$ real |
| Multiplication by $\cos(\omega_0 t)$ | $x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$ |
| Differentiation in time domain | $\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega)$   $n = 1, 2, \ldots$ |
| Integration | $\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$ |
| Convolution in time domain | $x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$ |
| Multiplication in time domain | $x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$ |
| Parseval's theorem | $\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$ |
| Parseval's theorem (special case) | $\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ |
| Duality | $X(t) \leftrightarrow 2\pi x(-\omega)$ |

## Common Fourier Transform Pairs

| $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$ | $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ |
|---|---|
| $1$   $(-\infty < t < \infty)$ | $2\pi\delta(\omega)$ |
| $-0.5 + u(t)$ | $\frac{1}{j\omega}$ |
| $u(t)$ | $\pi\delta(\omega) + \frac{1}{j\omega}$ |
| $\delta(t)$ | $1$ |
| $\delta(t - c)$ | $e^{-j\omega c}$   ($c$ any real number) |
| $e^{-bt}u(t)$ | $\frac{1}{j\omega + b}$   $(b > 0)$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0)$     ($\omega_0$ any real number) |
| $p_\tau(t)$ | $\tau\,\text{sinc}\frac{\tau\omega}{2\pi}$ |
| $\tau\,\text{sinc}\frac{\tau t}{2\pi}$ | $2\pi p_\tau(\omega)$ |
| $\left(1 - \frac{2|t|}{\tau}\right)p_\tau(t)$ | $\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$ |
| $\frac{\tau}{2}\text{sinc}^2\frac{\tau t}{4\pi}$ | $2\pi\left(1 - \frac{2|\omega|}{\tau}\right)p_\tau(\omega)$ |
| $\cos(\omega_0 t + \theta)$ | $\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$ |
| $\sin(\omega_0 t + \theta)$ | $j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$ |

## Trigonometric identities

$\sin(-\theta) = -\sin(\theta)$      $\cos(-\theta) = \cos(\theta)$      $\tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$      $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2)$      $\cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$