

# **EEE2035F EXAM SIGNALS AND SYSTEMS I**

University of Cape Town  
Department of Electrical Engineering

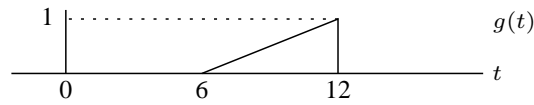
June 2012  
2 hours

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## **Information**

- The exam is closed-book.
  - There are 8 questions totalling 75 marks. You must answer all of them.
  - The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
  - You have 2 hours.
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1. Consider the signal  $g(t)$  below:



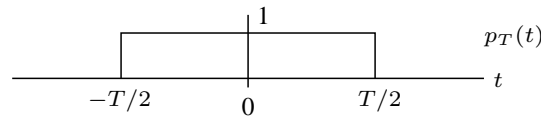
- (a) Sketch the generalised derivative of  $g(t)$ .
- (b) Sketch  $x(t) = \int_{-\infty}^t g(\tau) d\tau$ .
- (c) Sketch the signal  $y(t) = g(4 - 2t)$ . Show all the steps.

(10 marks)

2. Find the convolution of the two signals

$$x(t) = 2p_1(t - 3/2) \quad \text{and} \quad h(t) = -p_2(t - 1),$$

where  $p_T(t)$  is defined as follows:



(10 marks)

3. A system is defined by the relationship

$$y(t) = x(2t),$$

where  $x(t)$  is the input and  $y(t)$  is the output.

- (a) Is the system causal?
- (b) Find and plot the output when the input is  $x_1(t) = u(t)$ .
- (c) Find and plot the output when the input is  $x_2(t) = u(t - 1)$ .
- (d) Is the system linear and time invariant?

(10 marks)

4. (a) Find the Fourier transform of  $g(t) = te^{-2t}u(t)$ .

(b) Suppose

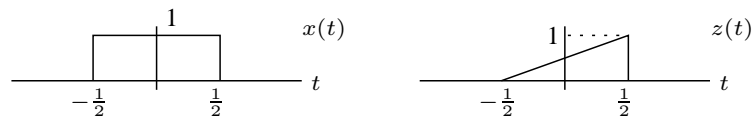
$$x(t) = 2e^{-2t}u(t) \quad \text{and} \quad h(t) = e^{-4t}u(t).$$

Find the Fourier transform of the above signals and use your result to calculate a time-domain expression for  $y(t) = x(t) * h(t)$ .

(10 marks)

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5. Consider two signals  $x(t)$  and  $z(t)$  shown below:



(a) From the direct definition of the Fourier transform as an integral equation, show that the transform of the signal  $x(t)$  is

$$X(\omega) = \frac{2 \sin(\omega/2)}{\omega}.$$

(b) Show that the Fourier transform of  $z(t)$  is

$$Z(\omega) = \frac{1}{j\omega} \left( \frac{2 \sin(\omega/2)}{\omega} - e^{-j\omega/2} \right).$$

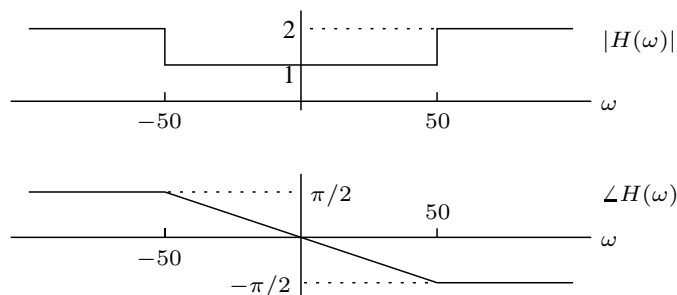
(Hint: it is quite easy to find the Fourier transform of the derivative of  $z(t)$ .)

(c) Suppose a system with impulse response  $z(t)$  is driven by an input  $x(t)$ . Find an expression for the output  $Y(\omega)$  in the frequency domain.

(10 marks)

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6. The signal  $x(t) = \cos(25t) + \cos(60t)$  is provided as input to a linear time-invariant system with the frequency response shown below:



- (a) Sketch the magnitude spectrum  $|X(\omega)|$  of the signal  $x(t)$ .  
 (b) The signal  $x(t)$  is periodic, and hence has a Fourier series representation of the form

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}.$$

Show that  $\omega_0 = 5$  radians/second and  $c_5 = c_{-5} = c_{12} = c_{-12} = \frac{1}{2}$  with all other coefficients zero determines the signal correctly.

- (c) Use Parseval's theorem to find the power of the signal  $x(t)$ .  
 (d) Determine the output signal  $y(t)$ .

(10 marks)

7. (a) Simplify the following expression:  $w(t) = \delta(t - 3) \frac{4 - jt^2}{2t}$ .  
 (b) Find the Fourier transform of  $x(t) = (\cos(5t) + e^{-2t})u(t)$ .  
 (c) Find the inverse Fourier transform of  $Y(\omega) = \frac{5\omega}{j\omega + 1}$ .

(10 marks)

8. Consider the signal

$$x(t) = \text{sinc}^2(t/2\pi).$$

- (a) Find and plot  $X(\omega)$ .
- (b) According to Nyquist, what is the largest sampling interval  $T$  for which the signal can be reconstructed from samples?
- (c) Sketch the spectrum of  $x_s(t) = x(t)p(t)$ , where

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

and  $T_s = \frac{2}{3}\pi$ .

(5 marks)

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# INFORMATION SHEET

## Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of $t$	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \dots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0 \text{ real}$
Multiplication by $\cos(\omega_0 t)$	$x(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \dots$
Integration	$\int_{-\infty}^t x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

## Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$
1 $(-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t - c)$	$e^{-j\omega c} \quad (c \text{ any real number})$
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad (\omega_0 \text{ any real number})$
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\tau \text{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2 t }{\tau}\right) p_\tau(t)$	$\frac{\tau}{2} \text{sinc}^2 \left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2} \text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi \left(1 - \frac{2 \omega }{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

## Trigonometric identities

$$\begin{aligned}
 \sin(-\theta) &= -\sin(\theta) & \cos(-\theta) &= \cos(\theta) & \tan(-\theta) &= -\tan(\theta) \\
 \sin^2(\theta) + \cos^2(\theta) &= 1 & \sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\
 \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta) \\
 \sin(\theta_1 + \theta_2) &= \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) & \cos(\theta_1 + \theta_2) &= \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) \\
 e^{j\theta} &= \cos(\theta) + j\sin(\theta)
 \end{aligned}$$