EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2011

1. *Most of these are routine. For part (c), note that the sifting property implies that*

$$y_3(t) = \int_{-\infty}^{\infty} x(t+1)\delta(t-\lambda)d\lambda = x(t+1).$$

For part (d), because the signal is zero for negative time, the signal $y_4(t)$ is just the indefinite integral of $x(t)$ (i.e. the lower limit of zero can be changed to $-\infty$ without modifying the result. For part (e) you can plot the signal $x(t-1)$ and then take the generalised derivative (including the impulse in the result), or alternatively just take the derivative of $x(t)$ and shift — the order of operations doesn't matter because shifting and differentiating are both linear operations.

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2. *For the first two parts substitute and use the sifting property to find (a) $y(t) = g(t-2)$ and (b) $y(t) = g(t-4)$. The two inputs differ by a shift of one unit, while the outputs differ by a shift of two, so the system is not time invariant (and therefore not LTI).*

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3. *The output is $y(t) = h(t) * x(t)$. Using the derivative property $y'(t) = h'(t) * x(t)$ (where prime denotes derivative). The signal $h'(t)$ is just two delta functions, so the convolution is simple and one can easily find $y'(t)$. Integrate (indefinite) to get the required result.*

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4. *You can use first principles to calculate the Fourier series coefficients, although it is tricky to get the result into a symmetric form where the magnitude and phase can be easily obtained. More simply, using the given series representation along with the observed relationship $y(t) = 2x(t-1/2)$ a useful form for the coefficients can be obtained directly.*

5. *The frequency response is the Fourier transform of the impulse response, and can be seen to be $H(\omega) = 2\pi p_3(\omega)$. This is an ideal lowpass filter that passes frequencies $|\omega| \leq 3/2$ (with a gain of 2π) and eliminates the rest. Thus the response to $x_1(t)$ will be $2\pi \cos(t)$, and the response to $x_2(t)$ will be zero. These results can be formally shown by noting that a complex exponential $e^{j\omega_0 t}$ into a linear system produces the output $H(\omega_0)e^{j\omega_0 t}$. Else consider plots of the signals in the frequency domain use multiplication to get the required results.*

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6. *The input-output relationship for a linear system in frequency coordinates is $Y(\omega) = H(\omega)X(\omega)$, so the frequency response is $H(\omega) = Y(\omega)/X(\omega)$ after finding $Y(\omega)$ and $X(\omega)$. The impulse response is the inverse Fourier transform of $H(\omega)$, which can be found using a partial fraction expansion.*

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7. *The result for (a) can be found by applying the time shift property to the signal $e^{-3t}u(t)$ and manipulating accordingly. For part (b), ignore the differentiation operation initially, and use the Euler expansion for $\sin$ to find the transform by applying the frequency translation property to the two parts of the signal. Differentiation in time is then just multiplication by $j\omega$.*

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8. *Multiplication by a cosine corresponds to modulation or frequency domain shifting. The signal $V(\omega)$ contains two replicas of $X(\omega)$, one at $\omega = 4$ and one at $\omega = -4$, both with height π. The signal $z(t)$ is the same operation but instead applied to $V(\omega)$, and contains two replicas of $X(\omega)$ (each of height π^2) at $\omega = \pm 8$, and one of height $2\pi^2$ at DC ($\omega = 0$). To reconstruct $x(t)$ at the output the filter should pass this central replica and scale it to have unit height. The cutoff is variable but must be in the range $2 \leq \omega \leq 6$.*
