EEE2035F EXAM
SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2011
2 hours

Information

- The exam is closed-book.
- There are *eight* questions totalling 80 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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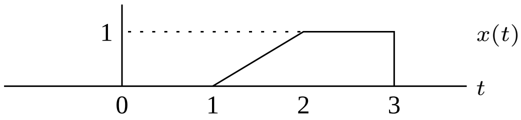
1. Suppose $x(t)$ is the signal below:



Sketch the following:

(a) $y_1(t) = -2x(t) + 1$

(b) $y_2(t) = x(-t + 3)$

(c) $y_3(t) = \int_{-\infty}^{\infty} x(\lambda + 1)\delta(t - \lambda)d\lambda$

(d) $y_4(t) = \int_0^t x(\lambda)d\lambda$

(e) $y_5(t) = \frac{d}{dt}x(t - 1)$

(f) $y_6(t) = x(2t - 1)$.

(15 marks)

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2. A linear system has the relationship

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)g(t - 2\lambda)d\lambda$$

between its input $x(t)$ and its output $y(t)$, where $g(t) = u(t) - u(t - 4)$.

(a) Determine $y(t)$ when $x(t) = \delta(t - 1)$.

(b) Determine $y(t)$ when $x(t) = \delta(t - 2)$.

(c) Use your answers for (a) and (b) to determine whether the system is LTI.

(10 marks)

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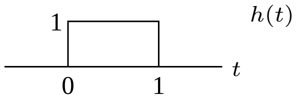
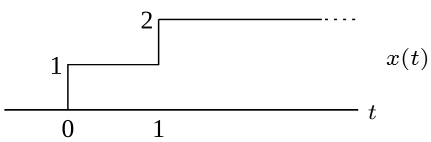
3. A system with impulse response $h(t)$ is driven by the input signal $x(t)$, both of which are shown below:





Find and plot the output signal $y(t)$.

(10 marks)

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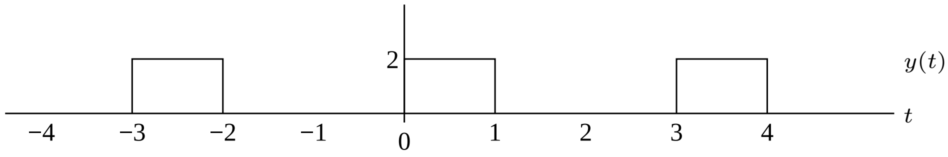
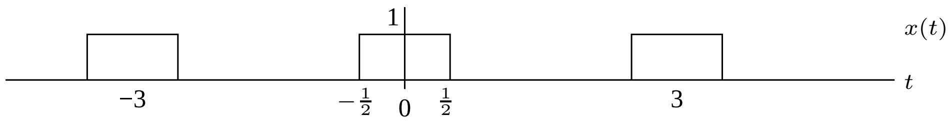
4. Find the Fourier series representation of the periodic signal $y(t)$ shown below:



Plot the magnitude and phase of the coefficients over the range $k = -3, \ldots, 3$.

You may choose to use the fact that the periodic signal



has a Fourier series expansion $x(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\frac{2\pi}{3}t}$ with $d_k = \frac{1}{k\pi}\sin(k\pi/3)$ and $d_0 = 1/3$.

(10 marks)

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5. A system has the following impulse response:

$$h(t) = 3\,\text{sinc}\left(\frac{3t}{2\pi}\right).$$

(a) Sketch the frequency response $H(\omega)$ of the system.

(b) What is the response $y_1(t)$ to the input signal $x_1(t) = \cos(t)$?

(c) What is the response $y_2(t)$ to the input signal $x_2(t) = \cos(2t)$?

(10 marks)

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6. If the output of an LTI system is $y(t) = 3e^{-5t}u(t) - e^{-2t}u(t)$ when the input is $x(t) = e^{-5t}u(t)$, what is the impulse response $h(t)$?

(10 marks)

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7. (a) Use tables of Fourier transforms and properties to show that the following is a valid Fourier pair:

$$e^{-3t}u(t-1) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad e^{-3}e^{-j\omega}\frac{1}{3+j\omega}.$$

(b) Use this result along with other Fourier transform properties to determine the spectrum of the signal

$$x(t) = \frac{d}{dt}(\sin(2t)e^{-3t}u(t-1)).$$

(10 marks)

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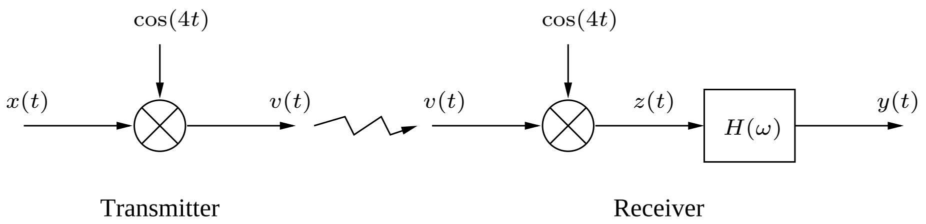
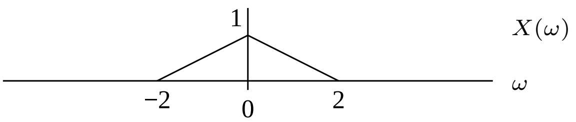
8. A block diagram of a transmitter and receiver is shown below:



The input signal is assumed to have the following spectrum:



(a) Sketch the signals $V(\omega)$ and $Z(\omega)$.

(b) Specify the frequency response $H(\omega)$ of the filter required for the output $y(t)$ to be the same as the input $x(t)$.

(5 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\tau \text{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega+\omega_0) + e^{j\theta}\delta(\omega-\omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega+\omega_0) - e^{j\theta}\delta(\omega-\omega_0)]$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$