## EEE2035F EXAM SIGNALS AND SYSTEMS I

## HINTS: June 2010

- 1. (a) Since  $x_1(t) = x(-(t-1)) = x'_1(t-1)$  with  $x'_1(t) = x_1(-t)$ , so flip around origin and shift result to the right (delay) by 1 unit.
  - (b) Delay (shift right) by 1 and change sign of range values.
  - (c) Indefinite integral: zero for  $t \le -2$ , quadratic (concave upwards) over range -2 to 0  $(x_3(0) = 2)$ , linear decrease to  $x_3(1) = 1$ , then constant  $x_3(t) = 1$  for  $t \ge 1$ .
  - (d) Sifting property:  $x_4(t) = \delta(t+1)x(-1) = \delta(t+1)$ , so just sketch an impulse.
  - (e) Generalised derivative is ordinary derivative (only nonzero slope is 1 over range -2 to 0), with impulses of size -3 at t = 0 and 1 at t = 1 to handle discontinuities.
- 2. (a) Impulse response is output y(t) when input is  $x(t) = \delta(t)$ , so must have

$$h(t) = \int_{t-1}^{t} \delta(\tau) d\tau = u(t) - u(t-1).$$

- (b) Impulse response h(t) is zero for t < 0 so system is causal.
- (c) Overall impulse response is  $h_o(t) = h(t) * h(t)$ , so get result by convolution (using any method that is easy for the problem). Result is a triangular pulse centered on t = 1, total width 2, and height 1.
- 3. (a) Easiest to convolve  $e^{-2t}u(t)$  with u(t) using derivative property, then delay the result by one time unit to get the answer.
  - (b) Since h(t) = 0 for t < 0 the system is causal.
- 4. (a) Take the Fourier transform of the differential equation and solve for the transfer

function to give

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{2+j\omega}.$$

One approach is to find the impulse response h(t) and get the solution by convolution with given x(t). Alternatively find  $X(\omega)$  from x(t) and use the inverse transform of  $Y(\omega) = H(\omega)X(\omega)$  (which will require partial fractions) to get  $y(t) = e^{-t}u(t) - e^{-2t}u(t)$ .

- (b) Could proceed as with the previous case, but it's easier to find h(t) and then convolve with u(t) using the differentiation property to find output  $y(t) = \frac{1}{2}(1 e^{-2t})u(t)$ .
- 5. (a) First find the transform of  $g_1(t) = e^{-\pi |t|} = e^{-\pi t}u(t) + e^{-\pi (-t)}u(-t)$ , then use the time shift property to give the required result.
  - (b) The inverse transform of

$$F_b(\omega) = \frac{6j\omega}{2+j\omega} = 6 - \frac{12}{2+j\omega}$$

is  $f_b(t) = 6\delta(t) - 12e^{-2t}u(t)$ . Applying frequency shift yields the required inverse  $f(t) = 6\delta(t) - 12e^{-(2+j)t}u(t)$ .

- (c) Convolution in time is multiplication in frequency, so the result can be found as  $M(\omega) = 2 \operatorname{sinc}(\omega/\pi) p_{2\pi}(\omega).$
- 6. The signal x(t) has period T = 4 and using the integration formula the coefficients of the Fourier series can be found to be  $c_k = 2j \sin(k\omega_0)$  with  $\omega_0 = 2\pi/4$ . Since

$$x(t) = \frac{d}{dt}y(t) = \sum_{k=-\infty}^{\infty} d_k \frac{d}{dt} e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} (jk\omega_0 d_k) e^{jk\omega_0 t}$$

we have  $c_k = jk\omega_0 d_k$ , so  $d_k = \frac{2}{k\omega_0} \sin(k\omega_0)$  for  $k \neq 0$  and  $d_0 = 1/2$ .

- 7. (a) If x(t) = 1 then  $V(\omega) = \pi(\delta(\omega + 5) + \delta(\omega 5))$ , which doesn't lie in the passband of the filter. Thus w(t) = 0 and y(t) = 0.
  - (b) In this case  $V(\omega) = \pi(\delta(\omega) + \delta(\omega 10))$  and  $W(\omega) = \pi(\delta(\omega))$ . Thus

$$Y(\omega) = \frac{\pi}{2}(\delta(\omega+5) + \delta(\omega-5)), \text{ so } y(t) = \cos(5t).$$

- 8. (a) The convolution yields a signal that contains replicas of  $X(\omega)$  centered at integer multiples of 15.
  - (b) Use a lowpass filter (ideal) with a cutoff frequency between 5 and 10 radians/second.