EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2010

1. (a) *Since $x_1(t) = x(-(t-1)) = x_1'(t-1)$ with $x_1'(t) = x_1(-t)$, so flip around origin and shift result to the right (delay) by 1 unit.*

   (b) *Delay (shift right) by 1 and change sign of range values.*

   (c) *Indefinite integral: zero for $t \leq -2$, quadratic (concave upwards) over range -2 to 0 ($x_3(0) = 2$), linear decrease to $x_3(1) = 1$, then constant $x_3(t) = 1$ for $t \geq 1$.*

   (d) *Sifting property: $x_4(t) = \delta(t+1)x(-1) = \delta(t+1)$, so just sketch an impulse.*

   (e) *Generalised derivative is ordinary derivative (only nonzero slope is 1 over range -2 to 0), with impulses of size -3 at $t = 0$ and 1 at $t = 1$ to handle discontinuities.*

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2. (a) *Impulse response is output $y(t)$ when input is $x(t) = \delta(t)$, so must have*

$$h(t) = \int_{t-1}^{t} \delta(\tau) d\tau = u(t) - u(t-1).$$

   (b) *Impulse response $h(t)$ is zero for $t < 0$ so system is causal.*

   (c) *Overall impulse response is $h_o(t) = h(t) * h(t)$, so get result by convolution (using any method that is easy for the problem). Result is a triangular pulse centered on $t = 1$, total width 2, and height 1.*

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3. (a) *Easiest to convolve $e^{-2t}u(t)$ with $u(t)$ using derivative property, then delay the result by one time unit to get the answer.*

   (b) *Since $h(t) = 0$ for $t < 0$ the system is causal.*

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4. (a) *Take the Fourier transform of the differential equation and solve for the transfer*

function to give

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{2 + j\omega}.$$

One approach is to find the impulse response $h(t)$ and get the solution by convolution with given $x(t)$. Alternatively find $X(\omega)$ from $x(t)$ and use the inverse transform of $Y(\omega) = H(\omega)X(\omega)$ (which will require partial fractions) to get $y(t) = e^{-t}u(t) - e^{-2t}u(t)$.

(b) *Could proceed as with the previous case, but it's easier to find $h(t)$ and then convolve with $u(t)$ using the differentiation property to find output $y(t) = \frac{1}{2}(1 - e^{-2t})u(t)$.*

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5. (a) *First find the transform of $g_1(t) = e^{-\pi|t|} = e^{-\pi t}u(t) + e^{-\pi(-t)}u(-t)$, then use the time shift property to give the required result.*

   (b) *The inverse transform of*

$$F_b(\omega) = \frac{6j\omega}{2 + j\omega} = 6 - \frac{12}{2 + j\omega}$$

   is $f_b(t) = 6\delta(t) - 12e^{-2t}u(t)$. Applying frequency shift yields the required inverse $f(t) = 6\delta(t) - 12e^{-(2+j)t}u(t)$.

   (c) *Convolution in time is multiplication in frequency, so the result can be found as $M(\omega) = 2sinc(\omega/\pi)p_{2\pi}(\omega)$.*

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6. *The signal $x(t)$ has period $T = 4$ and using the integration formula the coefficients of the Fourier series can be found to be $c_k = 2j\sin(k\omega_0)$ with $\omega_0 = 2\pi/4$. Since*

$$x(t) = \frac{d}{dt}y(t) = \sum_{k=-\infty}^{\infty} d_k \frac{d}{dt}e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} (jk\omega_0 d_k)e^{jk\omega_0 t}$$

   we have $c_k = jk\omega_0 d_k$, so $d_k = \frac{2}{k\omega_0}\sin(k\omega_0)$ for $k \neq 0$ and $d_0 = 1/2$.

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7. (a) *If $x(t) = 1$ then $V(\omega) = \pi(\delta(\omega+5) + \delta(\omega-5))$, which doesn't lie in the passband of the filter. Thus $w(t) = 0$ and $y(t) = 0$.*

   (b) *In this case $V(\omega) = \pi(\delta(\omega) + \delta(\omega - 10))$ and $W(\omega) = \pi(\delta(\omega)$. Thus*

$Y(\omega) = \frac{\pi}{2}(\delta(\omega+5) + \delta(\omega-5))$, *so* $y(t) = \cos(5t)$.

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8. (a) *The convolution yields a signal that contains replicas of* $X(\omega)$ *centered at integer multiples of 15.*

   (b) *Use a lowpass filter (ideal) with a cutoff frequency between 5 and 10 radians/second.*
