EEE2035F EXAM
SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2010
2 hours

Information

- The exam is closed-book.
- There are *eight* questions totalling 80 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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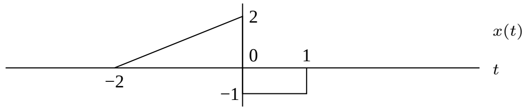
1. Let $x(t)$ be the signal below:



Sketch the following:

(a) $x_1(t) = x(1-t)$

(b) $x_2(t) = -x(t-1)$

(c) $x_3(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(d) $x_4(t) = \delta(t+1)x(t)$

(e) $x_5(t) = \frac{d}{dt}x(t)$.

(10 marks)

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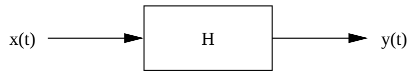
2. Consider the system



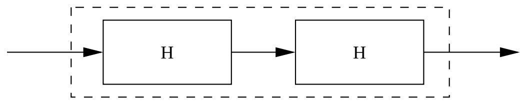
with input-output relationship

$$y(t) = \int_{t-1}^{t} x(\tau)d\tau$$

(a) Assuming that the system is linear and time invariant, show that the impulse response is $h(t) = u(t) - u(t-1)$.

(b) Is the system causal? Why?

(c) Find and sketch the impulse response of the cascaded system below:



(10 marks)

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3. A linear time-invariant system has an impulse response of $h(t) = e^{-2(t-1)}u(t-1)$.

(a) Use time-domain convolution to find and sketch the output $y(t)$ when the input is $x(t) = u(t)$.

(b) Is the system causal? Why?

(10 marks)

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4. Consider a continuous-time LTI system described by

$$\frac{dy(t)}{dt} + 2y(t) = x(t),$$

where $x(t)$ is the input and $y(t)$ is the output. Using the Fourier transform and assuming initial rest conditions, find the output $y(t)$ for each of the following input signals:

(a) $x(t) = e^{-t}u(t)$,

(b) $x(t) = u(t)$.

(10 marks)

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5. (a) Find the Fourier transform of the signal $g(t) = e^{-\pi|t-2|}$.

(b) Find the inverse Fourier transform of $F(\omega) = \frac{6j(\omega+1)}{2+j(\omega+1)}$.

(c) Find the Fourier transform of $m(t) = p_2(t) * \text{sinc}(t)$.

(15 marks)

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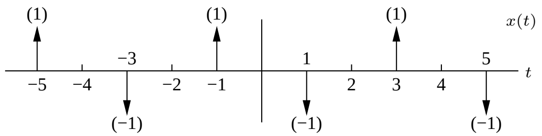
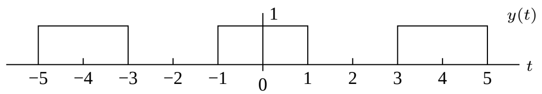
6. Find the Fourier series representation for the periodic signal below:



Use this result to find the Fourier series representation of



(10 marks)

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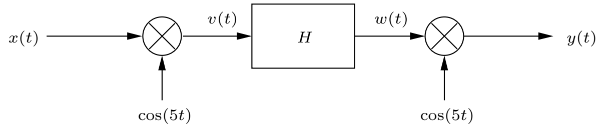
7. Consider the system shown below



where H is an ideal lowpass filter with frequency response $H(\omega) = p_2(\omega)$.

(a) What is the output $y(t)$ of the system if the input is the constant DC signal $x(t) = 1$?

(b) What is the output $y(t)$ of the system if the input is the complex exponential $x(t) = e^{j5t}$?

(10 marks)

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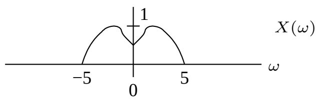
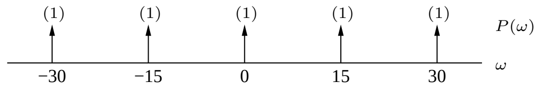
8. Let $X(\omega)$ be the signal



(a) If $P(\omega)$ is the impulse train $P(\omega) = \sum_{k=-\infty}^{\infty} \delta(\omega - 15k)$, shown below,



then sketch $Y(\omega) = X(\omega) * P(\omega)$.

(b) Can $X(\omega)$ be recovered from $Y(\omega)$? If so, how?

(5 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \,\text{sinc}\, \frac{\tau\omega}{2\pi}$
$\tau \,\text{sinc}\, \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$