EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2009

1. *Fundamental. Note that for part (c) the independent variable is λ (and t is a constant) — think the same operation that you apply in convolution. Don't forget the delta functions in (d) at the points of discontinuity (-3, -1, and 2). The solution to (e) is the indefinite integral, with an offset of 1 added afterwards.*

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2. *For part (a) see that $y(t) = h_2(t) * x(t)$ and $z(t) = h_1(t) * x(t)$, so $y(t) = h_2(t) * h_1(t) * x(t)$. Because convolution is associative and commutative the effective impulse response is therefore $h_2(t) * h_1(t) = h_1(t) * h_2(t)$. For part (b) it is easiest to just do convolution in the time domain, since convolution by a unit step is just indefinite integration.*

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3. *The important thing to note is that the differential equation can be Fourier transformed to $(2 + j\omega)Y(\omega) = X(\omega)$, so the transfer function of the system is $H(\omega) = Y(\omega)/X(\omega) = 1/(2 + j\omega)$. For part (a) find $X(\omega)$ and calculate $Y(\omega) = H(\omega)X(\omega)$ to get the required result in the frequency domain. Use partial fractions to invert to $y(t)$ (although the question doesn't specify that a time-domain result is required). Part (b) can be done similarly, although it's probably easier to note that $h(t) = e^{-2t}u(t)$ and do time-domain convolution with the given $x(t)$ — because it's a unit step, the output will just be the indefinite integral of $h(t)$.*

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4. *The output of a LTI system with complex exponential input $e^{j\omega_0 t}$ is $H(\omega_0)e^{j\omega_0 t}$. The output for (a) is therefore simply $H(5)e^{j5t}$, and $H(5)$ can be obtained from the Fourier transform of $h(t)$. For part (b) the Euler expansion can be used to split the $\cos$ into two complex exponentials and the same method applied as for part (a). You should be able to*

write the result as a real-valued cosine with a little effort.

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5. *Sinusoids are already almost in the form of a Fourier series. Apply Euler's expansion to the right-hand side and equate terms to get the required coefficients. Note that the fundamental frequency should be chosen as* $\omega_0 = 3/14$ *radians per second. The Fourier transform can be applied to the expression for the Fourier series to answer the second part of the question, using the fact that* $e^{j\omega_0 t} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta(\omega - \omega_0)$.

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6. *For part (a), plotting* $H(\omega)$ *you'll see a rect function around* $\omega = -2.5$ *and* $\omega = 2.5$*. This can be considered the result of applying modulation (multiplication) by a cosine to the signal that is a rect function around the origin in the frequency domain — a sinc function in time. The solution is therefore a sinc multiplied by a cosine. For part (b) it is easy to see that* $y(t) = 2h(t-1) - 3h(t-2)$*, and an expression for the result follows from the solution to (a).*

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7. *For part (a), observe that* $H(\omega)$ *is already in magnitude and phase form, with* $|H(\omega)| = 2$ *and* $\angle H(\omega) = -2\omega$*. Add or subtract multiples of* 2π *in the phase plot wherever it falls outside of the specified range. For part (b) the impulse response is the inverse Fourier transform of* $H(\omega)$*, and is just be a delta function of size 2 at* $t = 2$*. Because the impulse response is right sided the system is causal.*

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8. *Multiplication in time corresponds to convolution in frequency. The sinc part transforms to a rect, and the cosine transforms to two impulses, so the required transform contains two rect functions at* $\omega = \pm 10\pi$*. The sampling theorem states that the signal must be sampled at twice the highest frequency present, which can be found from the sketch of* $X(\omega)$*. Sampling corresponds to multiplication by an impulse train in time, or convolution with an impulse train (spacing* ω_s*) in frequency. Solutions for parts (c) and (d) follow accordingly.*
