EEE2035F EXAM
SIGNALS AND SYSTEMS I

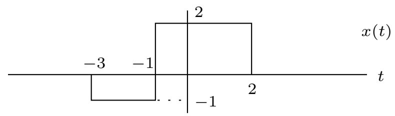
University of Cape Town
Department of Electrical Engineering

June 2009
2 hours

Information

- The exam is closed-book.
- There are *eight* questions totalling 80 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

1. Consider the signal $x(t)$ below:



Plot the following:

(a) $x_1(t) = -x(-t)$

(b) $x_2(t) = x(2t-2)$

(c) $x_3(\lambda) = x(t-\lambda)$

(d) $x_4(t) = \frac{d}{dt}x(t)$

(e) $x_5(t) = \int_{-\infty}^{t} x(\lambda)d\lambda + 1$

(10 marks)

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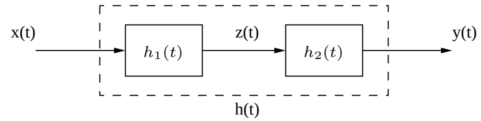
2. Two systems are cascaded as shown below:



The impulse responses of the individual systems are $h_1(t) = u(t)$ and $h_2(t) = e^{-t}u(t)$.

(a) Show that the impulse response of the combined system linking the input $x(t)$ with the output $y(t)$ is $h(t) = h_1(t) * h_2(t)$.

(b) Use time-domain convolution to find $h(t)$.

(10 marks)

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3. Consider a continuous-time LTI system described by

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

(a) Use frequency domain multiplication to find the output of the system when the input is $x(t) = e^{-t}u(t)$.

(b) Use a method of your choice to find the output of the system when the input is $x(t) = u(t)$.

(10 marks)

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4. Find the output of the system with impulse response

$$h(t) = e^{-3t}u(t)$$

for each of the following inputs:

(a) $x(t) = e^{j5t}$

(b) $x(t) = 3\cos(5t)$.

(10 marks)

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5. Find the Fourier coefficients c_k and the fundamental frequency ω_0 such that

$$\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} = -2 + 4\sin\left(\frac{3t}{7}\right) - 3\cos\left(\frac{3t}{14}\right).$$

Use this result to find and plot the magnitude $|X(\omega)|$ of the Fourier transform of the signal $x(t) = -2 + 4\sin\left(\frac{3t}{7}\right) - 3\cos\left(\frac{3t}{14}\right)$.

(10 marks)

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6. (a) Compute the inverse Fourier transform $h(t)$ of the function

$$H(\omega) = \begin{cases} 2 & -3 \leq \omega \leq -2 \text{ or } 2 \leq \omega \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

(b) If $x(t) = 2\delta(t-1) - 3\delta(t-2)$, find $y(t) = h(t) * x(t)$.

(10 marks)

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7. A system has a Frequency response given by $H(\omega) = 2e^{-j2\omega}$.

(a) Sketch the magnitude and phase of $H(\omega)$. The vertical range of your phase plot should be from $-\pi$ to π. Label your plots clearly.

(b) What is the impulse response of the system?

(c) Is the system causal? Why?

(10 marks)

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8. A signal is given by

$$x(t) = \cos(10\pi t)\left(\frac{\sin(\pi t)}{t}\right).$$

(a) Find an expression for the Fourier transform $X(\omega)$, and sketch it.

(b) According to Nyquist, what is the smallest sampling frequency that can be used to sample $x(t)$ so that full reconstruction is possible?

(c) If you sample at $\omega_s = 15\pi$ rad/s, what is the spectrum of $x_s(t)$?

(d) If you sample at $\omega_s = 30\pi$ rad/s, what is the spectrum of $x_s(t)$?

(10 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \operatorname{sinc} \frac{\tau\omega}{2\pi}$
$\tau \operatorname{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\operatorname{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$