# EEE2035F EXAM SIGNALS AND SYSTEMS I

University of Cape Town Department of Electrical Engineering

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2 hours

#### 1. Consider the signal

Sketch the following: (a)  $y_1(t) = x(t)$ (b)  $y_2(t) = x(t-2)$ (c)  $y_3(t) = x(2-t)$ (d)  $y_4(t) = -x(-2t)$ (e)  $y_5(t) = x(t)\delta(t - \frac{1}{2})$ (f)  $y_6(t) = \int_{-\infty}^t x(\lambda)d\lambda$ (g)  $y_7(t) = \frac{dx(t)}{dt}$  (generalised).

(10 marks)

2. A system is described by the input/output relationship

y(t) = x(t/2).

x(t) = tu(t) - tu(t-2).

(a) Find the output  $y_1(t)$  when the input is  $x_1(t) = u(t)$ .

(b) Find the output  $y_2(t)$  when the input is  $x_2(t) = u(t-1)$ .

(c) What values of x(t) do you have to know to find y(-1)?

(d) Is the system linear? Why?

(e) Is the system time-invariant? Why?

(f) Is the system causal? Why?

#### (10 marks)

3. A system has an impulse response

 $h(t) = 3e^{-10t}u(t)$ 

Find the response of the system to a unit step input.

(10 marks)

### Information

- The exam is closed-book.
- There are *eight* questions totalling 80 marks. You must answer all of them.
- The page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

4. Consider the periodic function x(t) below:



- (a) Find the complex exponential Fourier series representation of x(t), along with a closed-form expression for the coefficients, and
- (b) Find values for the magnitude and phase of the coefficients corresponding to the second harmonic.

(10 marks)

5. Consider the signal  $x(t) = e^{-|t|}$ .

(a) Sketch x(t).

(b) Show that the Fourier transform of x(t) is

$$X(\omega) = \frac{1}{j\omega+1} + \frac{1}{-j\omega+1} = \frac{2}{\omega^2+1}$$

(c) Find an expression for the Fourier transform of the modulated signal  $y(t) = x(t) \cos(10t)$ .

(10 marks)

6. Find the inverse Fourier transform of each spectrum below:

(a)  $X_1(\omega) = \frac{5}{(j\omega+3)^2}$ , using the Fourier transform pair  $e^{-at}u(t) \iff \frac{1}{j\omega+a}$  (a > 0) coupled with the multiplication by power of t property.

(b) 
$$X_2(\omega) = \frac{5\omega}{\omega^2 + 4}$$
, using the Fourier transform pair  $e^{-|t|} \iff \frac{2}{\omega^2 + 1}$ 

(10 marks)

7. The input and output of the circuit



can be shown to obey the relationship

$$x(t) = RC\frac{dy(t)}{dt} + y(t).$$

Assuming that  $R = 2M\Omega$  and  $C = 1\mu F$ ,

- (a) Find the frequency response of the circuit.
- (b) Find the response of the circuit to the input signal  $x(t) = \sin(2\pi t)$ .

(10 marks)

8. An LTI system is described by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}.$$

(a) Show that the frequency response of the system is given by

 $H(\omega) = \frac{j\omega + 1}{j\omega + 2}.$ 

(b) Find the impulse response of the system.

(10 marks)

## **INFORMATION SHEET**

# Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a})  a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of $t$	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega)  n = 1, 2, \dots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)  \omega_0 \text{ real}$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega)  n = 1, 2, \dots$
Integration	$\int_{-\infty}^{t} x(\lambda) d\lambda \leftrightarrow \frac{1}{i\omega} X(\omega) + \pi X(0) \delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

# **Common Fourier Transform Pairs**

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$
$1  (-\infty < t < \infty)$	$2\pi\delta(\omega)$
-0.5 + u(t)	$\frac{1}{j\omega}$
u(t)	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c}$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega+b}  (b>0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$ ( $\omega_0$ any real number)
$p_{\tau}(t)$	$\tau \operatorname{sinc} \frac{\tau \omega}{2\pi}$
$\tau \operatorname{sinc} \frac{\tau t}{2\pi}$	$2\pi p_{\tau}(\omega)$
$\left(1-\frac{2 t }{\tau}\right)p_{\tau}(t)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\operatorname{sinc}^2\frac{\tau t}{4\pi}$	$2\pi \left(1 - \frac{2 \omega }{\tau}\right) p_{\tau}(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi [e^{-j\theta}\delta(\omega+\omega_0) + e^{j\theta}\delta(\omega-\omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega+\omega_0)-e^{j\theta}\delta(\omega-\omega_0)]$

# **Trigonometric identities**

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\begin{split} \sin(-\theta) &= -\sin(\theta) & \cos(-\theta) = \cos(\theta) & \tan(-\theta) = -\tan(\theta) \\ \sin^2(\theta) + \cos^2(\theta) &= 1 & \sin(2\theta) = 2\sin(\theta)\cos(\theta) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta) \\ \sin(\theta_1 + \theta_2) &= \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) & \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) \\ e^{j\theta} &= \cos(\theta) + j\sin(\theta) \end{split}
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