# EEE2035F EXAM
# SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2008
2 hours

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## Information

- The exam is closed-book.
- There are *eight* questions totalling 80 marks. You must answer all of them.
- The page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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1. Consider the signal

$$x(t) = tu(t) - tu(t-2).$$

Sketch the following:

(a) $y_1(t) = x(t)$

(b) $y_2(t) = x(t-2)$

(c) $y_3(t) = x(2-t)$

(d) $y_4(t) = -x(-2t)$

(e) $y_5(t) = x(t)\delta(t-\frac{1}{2})$

(f) $y_6(t) = \int_{-\infty}^{t} x(\lambda)d\lambda$

(g) $y_7(t) = \frac{dx(t)}{dt}$ (generalised).

(10 marks)

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2. A system is described by the input/output relationship

$$y(t) = x(t/2).$$

(a) Find the output $y_1(t)$ when the input is $x_1(t) = u(t)$.

(b) Find the output $y_2(t)$ when the input is $x_2(t) = u(t-1)$.

(c) What values of $x(t)$ do you have to know to find $y(-1)$?

(d) Is the system linear? Why?

(e) Is the system time-invariant? Why?

(f) Is the system causal? Why?

(10 marks)

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3. A system has an impulse response

$$h(t) = 3e^{-10t}u(t)$$

Find the response of the system to a unit step input.

(10 marks)

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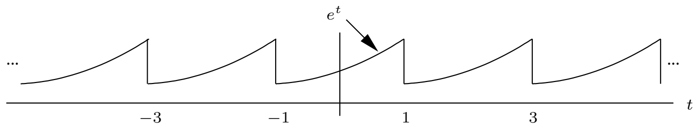
4. Consider the periodic function $x(t)$ below:




   (a) Find the complex exponential Fourier series representation of $x(t)$, along with a closed-form expression for the coefficients, and

   (b) Find values for the magnitude and phase of the coefficients corresponding to the second harmonic.

(10 marks)

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5. Consider the signal $x(t) = e^{-|t|}$.

   (a) Sketch $x(t)$.

   (b) Show that the Fourier transform of $x(t)$ is

$$X(\omega) = \frac{1}{j\omega + 1} + \frac{1}{-j\omega + 1} = \frac{2}{\omega^2 + 1}.$$

   (c) Find an expression for the Fourier transform of the modulated signal $y(t) = x(t)\cos(10t)$.

(10 marks)

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6. Find the inverse Fourier transform of each spectrum below:

   (a) $X_1(\omega) = \frac{5}{(j\omega+3)^2}$, using the Fourier transform pair $e^{-at}u(t) \Longleftrightarrow \frac{1}{j\omega+a}$ $(a > 0)$ coupled with the multiplication by power of $t$ property.

   (b) $X_2(\omega) = \frac{5\omega}{\omega^2+4}$, using the Fourier transform pair $e^{-|t|} \Longleftrightarrow \frac{2}{\omega^2+1}$.

(10 marks)

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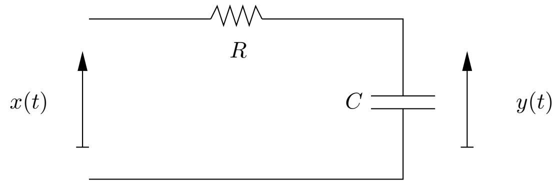
7. The input and output of the circuit




can be shown to obey the relationship

$$x(t) = RC\frac{dy(t)}{dt} + y(t).$$

Assuming that $R = 2M\Omega$ and $C = 1\mu F$,

   (a) Find the frequency response of the circuit.

   (b) Find the response of the circuit to the input signal $x(t) = \sin(2\pi t)$.

(10 marks)

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8. An LTI system is described by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}.$$

   (a) Show that the frequency response of the system is given by

$$H(\omega) = \frac{j\omega + 1}{j\omega + 2}.$$

   (b) Find the impulse response of the system.

(10 marks)

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# INFORMATION SHEET

## Fourier transform properties

| Property | Transform Pair/Property |
|---|---|
| Linearity | $ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$ |
| Time shift | $x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$ |
| Time scaling | $x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$ |
| Time reversal | $x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$ |
| Multiplication by power of $t$ | $t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$ |
| Frequency shift | $x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real |
| Multiplication by $\cos(\omega_0 t)$ | $x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega - \omega_0)]$ |
| Differentiation in time domain | $\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$ |
| Integration | $\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$ |
| Convolution in time domain | $x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$ |
| Multiplication in time domain | $x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$ |
| Parseval's theorem | $\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$ |
| Parseval's theorem (special case) | $\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$ |
| Duality | $X(t) \leftrightarrow 2\pi x(-\omega)$ |

## Common Fourier Transform Pairs

| $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$ | $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ |
|---|---|
| $1 \quad (-\infty < t < \infty)$ | $2\pi\delta(\omega)$ |
| $-0.5 + u(t)$ | $\frac{1}{j\omega}$ |
| $u(t)$ | $\pi\delta(\omega) + \frac{1}{j\omega}$ |
| $\delta(t)$ | $1$ |
| $\delta(t-c)$ | $e^{-j\omega c} \quad$ ($c$ any real number) |
| $e^{-bt}u(t)$ | $\frac{1}{j\omega + b} \quad (b > 0)$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0) \quad$ ($\omega_0$ any real number) |
| $p_\tau(t)$ | $\tau \,\text{sinc}\, \frac{\tau\omega}{2\pi}$ |
| $\tau \,\text{sinc}\, \frac{\tau t}{2\pi}$ | $2\pi p_\tau(\omega)$ |
| $\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$ | $\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$ |
| $\frac{\tau}{2}\text{sinc}^2\, \frac{\tau t}{4\pi}$ | $2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$ |
| $\cos(\omega_0 t + \theta)$ | $\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$ |
| $\sin(\omega_0 t + \theta)$ | $j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$ |

## Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$