EEE2035F EXAM
SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2008
2 hours

Information

- The exam is closed-book.
- There are *eight* questions totalling 80 marks. You must answer all of them.
- The page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

---

1. Consider the signal

$$x(t) = tu(t) - tu(t-2).$$

   Sketch the following:

   (a) $y_1(t) = x(t)$

   (b) $y_2(t) = x(t-2)$

   (c) $y_3(t) = x(2-t)$

   (d) $y_4(t) = -x(-2t)$

   (e) $y_5(t) = x(t)\delta(t - \frac{1}{2})$

   (f) $y_6(t) = \int_{-\infty}^{t} x(\lambda) d\lambda$

   (g) $y_7(t) = \frac{dx(t)}{dt}$ (generalised).

(10 marks)

---

2. A system is described by the input/output relationship

$$y(t) = x(t/2).$$

   (a) Find the output $y_1(t)$ when the input is $x_1(t) = u(t)$.

   (b) Find the output $y_2(t)$ when the input is $x_2(t) = u(t-1)$.

   (c) What values of $x(t)$ do you have to know to find $y(-1)$?

   (d) Is the system linear? Why?

   (e) Is the system time-invariant? Why?

   (f) Is the system causal? Why?

(10 marks)

---

3. A system has an impulse response

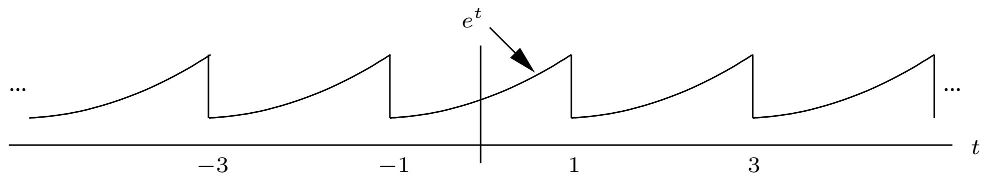
$$h(t) = 3e^{-10t}u(t)$$

   Find the response of the system to a unit step input.

(10 marks)

---

4. Consider the periodic function $x(t)$ below:



(a) Find the complex exponential Fourier series representation of $x(t)$, along with a closed-form expression for the coefficients, and

(b) Find values for the magnitude and phase of the coefficients corresponding to the second harmonic.

(10 marks)

---

5. Consider the signal $x(t) = e^{-|t|}$.

(a) Sketch $x(t)$.

(b) Show that the Fourier transform of $x(t)$ is

$$X(\omega) = \frac{1}{j\omega + 1} + \frac{1}{-j\omega + 1} = \frac{2}{\omega^2 + 1}.$$

(c) Find an expression for the Fourier transform of the modulated signal $y(t) = x(t)\cos(10t)$.

(10 marks)

---

6. Find the inverse Fourier transform of each spectrum below:

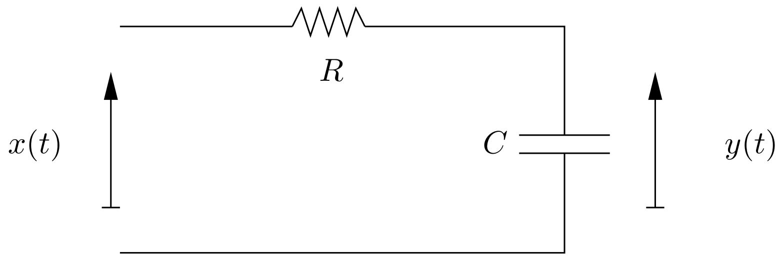
(a) $X_1(\omega) = \frac{5}{(j\omega+3)^2}$, using the Fourier transform pair $e^{-at}u(t) \Longleftrightarrow \frac{1}{j\omega+a}$ $(a > 0)$ coupled with the multiplication by power of t property.

(b) $X_2(\omega) = \frac{5\omega}{\omega^2+4}$, using the Fourier transform pair $e^{-|t|} \Longleftrightarrow \frac{2}{\omega^2+1}$.

(10 marks)

---

7. The input and output of the circuit



can be shown to obey the relationship

$$x(t) = RC\frac{dy(t)}{dt} + y(t).$$

Assuming that $R = 2M\Omega$ and $C = 1\mu F$,

(a) Find the frequency response of the circuit.

(b) Find the response of the circuit to the input signal $x(t) = \sin(2\pi t)$.

(10 marks)

---

8. An LTI system is described by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}.$$

(a) Show that the frequency response of the system is given by

$$H(\omega) = \frac{j\omega + 1}{j\omega + 2}.$$

(b) Find the impulse response of the system.

(10 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c}$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$ (ω_0 any real number)
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\tau \text{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega+\omega_0) + e^{j\theta}\delta(\omega-\omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega+\omega_0) - e^{j\theta}\delta(\omega-\omega_0)]$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$