

# EEE2035F EXAM SIGNALS AND SYSTEMS I

University of Cape Town  
Department of Electrical Engineering

June 2008

2 hours

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## Information

- The exam is closed-book.
  - There are *eight* questions totalling 80 marks. You must answer all of them.
  - The page of this exam paper contains an information sheet with standard Fourier transforms, transform properties, and some trigonometric identities.
  - You have 2 hours.
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1. Consider the signal

$$x(t) = tu(t) - tu(t - 2).$$

Sketch the following:

- (a)  $y_1(t) = x(t)$
- (b)  $y_2(t) = x(t - 2)$
- (c)  $y_3(t) = x(2 - t)$
- (d)  $y_4(t) = -x(-2t)$
- (e)  $y_5(t) = x(t)\delta(t - \frac{1}{2})$
- (f)  $y_6(t) = \int_{-\infty}^t x(\lambda)d\lambda$
- (g)  $y_7(t) = \frac{dx(t)}{dt}$  (generalised).

(10 marks)

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2. A system is described by the input/output relationship

$$y(t) = x(t/2).$$

- (a) Find the output  $y_1(t)$  when the input is  $x_1(t) = u(t)$ .
- (b) Find the output  $y_2(t)$  when the input is  $x_2(t) = u(t - 1)$ .
- (c) What values of  $x(t)$  do you have to know to find  $y(-1)$ ?
- (d) Is the system linear? Why?
- (e) Is the system time-invariant? Why?
- (f) Is the system causal? Why?

(10 marks)

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3. A system has an impulse response

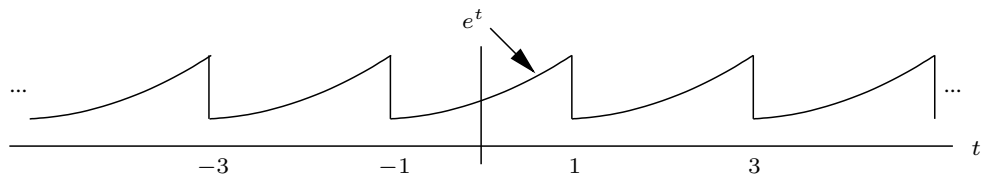
$$h(t) = 3e^{-10t}u(t)$$

Find the response of the system to a unit step input.

(10 marks)

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4. Consider the periodic function  $x(t)$  below:



- (a) Find the complex exponential Fourier series representation of  $x(t)$ , along with a closed-form expression for the coefficients, and
- (b) Find values for the magnitude and phase of the coefficients corresponding to the second harmonic.

(10 marks)

5. Consider the signal  $x(t) = e^{-|t|}$ .

- (a) Sketch  $x(t)$ .
- (b) Show that the Fourier transform of  $x(t)$  is

$$X(\omega) = \frac{1}{j\omega + 1} + \frac{1}{-j\omega + 1} = \frac{2}{\omega^2 + 1}.$$

- (c) Find an expression for the Fourier transform of the modulated signal  $y(t) = x(t) \cos(10t)$ .

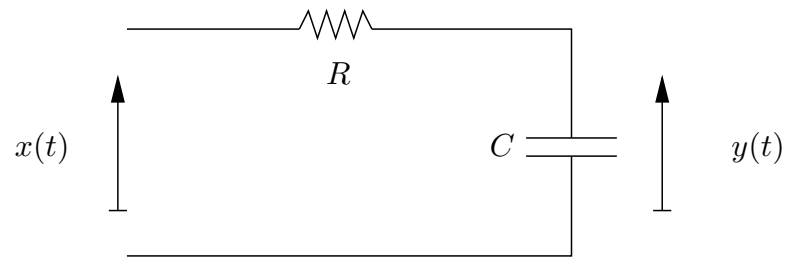
(10 marks)

6. Find the inverse Fourier transform of each spectrum below:

- (a)  $X_1(\omega) = \frac{5}{(j\omega+3)^2}$ , using the Fourier transform pair  $e^{-at}u(t) \iff \frac{1}{j\omega+a}$  ( $a > 0$ ) coupled with the multiplication by power of  $t$  property.
- (b)  $X_2(\omega) = \frac{5\omega}{\omega^2+4}$ , using the Fourier transform pair  $e^{-|t|} \iff \frac{2}{\omega^2+1}$ .

(10 marks)

7. The input and output of the circuit



can be shown to obey the relationship

$$x(t) = RC \frac{dy(t)}{dt} + y(t).$$

Assuming that  $R = 2M\Omega$  and  $C = 1\mu F$ ,

- (a) Find the frequency response of the circuit.
- (b) Find the response of the circuit to the input signal  $x(t) = \sin(2\pi t)$ .

(10 marks)

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8. An LTI system is described by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}.$$

- (a) Show that the frequency response of the system is given by

$$H(\omega) = \frac{j\omega + 1}{j\omega + 2}.$$

- (b) Find the impulse response of the system.

(10 marks)

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# INFORMATION SHEET

## Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{ a } X\left(\frac{\omega}{a}\right) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of $t$	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \dots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0 \text{ real}$
Multiplication by $\cos(\omega_0 t)$	$x(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \dots$
Integration	$\int_{-\infty}^t x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega) d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

## Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$
1 $(-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t - c)$	$e^{-j\omega c} \quad (c \text{ any real number})$
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad (\omega_0 \text{ any real number})$
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\tau \text{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2 t }{\tau}\right) p_\tau(t)$	$\frac{\tau}{2} \text{sinc}^2 \left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2} \text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi \left(1 - \frac{2 \omega }{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

## Trigonometric identities

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta)$$

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2 \cos^2(\theta) - 1 = 1 - 2 \sin^2(\theta)$$

$$\sin(\theta_1 + \theta_2) = \sin(\theta_1) \cos(\theta_2) + \cos(\theta_1) \sin(\theta_2) \quad \cos(\theta_1 + \theta_2) = \cos(\theta_1) \cos(\theta_2) - \sin(\theta_1) \sin(\theta_2)$$

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$