EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2007

1. *If you can't do this question then you're missing fundamental concepts — practice more. For part (f), note that for very large values of t you should have $y_6(t) = \int_{-\infty}^{\infty} g(\tau)d\tau = 1/2$, so the graph of the integral does not return to zero beyond $t = 2$. For part (g), the solution should have a dirac delta of size -1 at $t = 2$.*

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2. *You need to convolve the input $x(t)$ with the impulse response $h(t)$, and it's easiest to do this in the time domain. Note that the required response is*

$$y(t) = h(t) * x(t) = [\delta(t) + e^{-2t}u(t)] * x(t) = x(t) + e^{-2t}u(t) * x(t) = x(t) + y_1(t)$$

with $y_1(t) = e^{-2t}u(t) * x(t)$. This can be found directly, or by using the derivative property for convolution to turn the rectangular pulse into two deltas, colvolving, and integrating the result. One possible expression for the solution is

$$y(t) = p_2(t-1) + (1/2 - 1/2e^{-2t})u(t) - (1/2 - 1/2e^{-2(t-2)})u(t-2).$$

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3. *Need to evaluate the integral*

$$X(\omega) = \int_{-\infty}^{\infty} e^{-3t}u(t-2)e^{-j\omega t}dt$$

directly. Any good textbook will show how to do this for the slightly simpler signal $x(t) = e^{-bt}u(t)$ with $b > 0$. Solution is

$$X(\omega) = e^{-6}\frac{1}{(3+j\omega)}e^{-j\omega 2}.$$

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4. *Taking the Fourier transform of the expression and rearranging gives the frequency*

response

$$H(\omega) = \frac{j\omega}{\frac{1}{RC} + j\omega}.$$

The inverse transform is the impulse response. You will need the differentiation in time property to handle the $j\omega$ term in the numerator:

$$h(t) = \delta(t) - \frac{1}{RC} e^{-\frac{1}{RC}t} u(t).$$

This is a high-pass filter.

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5. *Since* $1 - (\frac{j\omega}{5}) = 1 + j(-\frac{\omega}{5}) = \sqrt{1 + (\omega/5)^2} e^{j \arctan(-\omega/5)}$ *the required magnitude is* $|H(\omega)| = 1/\sqrt{1 + \left(\frac{\omega}{5}\right)^2}$. *This is the transfer function of a low-pass filter. Since the response of the system to the input* $x_1(t) = e^{j\omega_0 t}$ *is*

$$\begin{aligned} y_1(t) &= \int_{-\infty}^{\infty} h(\lambda) x_1(t - \lambda) d\lambda = \int_{-\infty}^{\infty} h(\lambda) e^{j\omega_0 (t-\lambda)} d\lambda \\ &= \left(\int_{-\infty}^{\infty} h(\lambda) e^{-j\omega_0 \lambda} d\lambda \right) e^{j\omega_0 t} = H(\omega_0) e^{j\omega_0 t}, \end{aligned}$$

the response to $x(t) = \cos(3t) = 1/2 e^{j3t} + 1/2 e^{-j3t}$ *is* $y(t) = 1/2 H(3) e^{j3t} + 1/2 H(-3) e^{-j3t}$. *Noting that* $H(-3) = H^*(3)$ *the solution is found to be* $y(t) = 0.8575 \cos(3t + 0.5404)$.

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6. *Using tables and properties,* $F(\omega) = 2e^{-j\omega} + 2sinc(\omega/\pi) e^{j3\omega}$ *and* $G(\omega) = 2sinc(\omega/(2\pi)) sinc(\omega/\pi) e^{-j2\omega}$.

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7. *Since* $y(t) = x(t) e^{j5\pi t} = e^{j5\pi t} \sum_{k=-\infty}^{\infty} c_k e^{jk\pi t} = \sum_{k=-\infty}^{\infty} c_k e^{j(k+5)\pi t}$ *we can make the change of variables* $m = k + 5$ *to get* $y(t) = \sum_{m=-\infty}^{\infty} c_{m-5} e^{jm\pi t}$. *This is in the form of a Fourier series* $y(t) = \sum_{m=-\infty}^{\infty} d_m e^{jm\pi t}$ *with known coefficients* $d_m = c_{m-5}$. *The Fourier transform is* $Y(\omega) = \sum_{m=-\infty}^{\infty} c_{m-5} 2\pi \delta(\omega - m\pi)$.

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8. *Draw $x(t)$ and multiply it with $p(t)$: you should get a right-sided impulse train, with the size of each impulse equal to the value of $x(t)$ at the correponding point. Transforming the*

expression for $x_s(t)$ to the frequency domain gives $X_s(\omega) = 1/(2\pi)P(\omega) * X(\omega)$, which can be shown to equal $X_s(\omega) = 1/T \sum_{k=-\infty}^{\infty} X(\omega - k\omega_0)$. Since $X(\omega) = 1/(2 + j\omega)$ is not band limited, the terms in this sum overlap and the signal $X(\omega)$ cannot be recovered from $X_s(\omega)$. Thus $x(t)$ cannot be recovered from $x_s(t)$ — aliasing has occurred.
