EEE2035F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2006

1. *Fundamental stuff — practice if you're unsure. Note that* $y_6(t) = 3/2$ *for large values of* t*: the signal does not return to zero after* $t = 2$. *Remember the delta functions in* $y_7(t)$.

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2. *The system is linear — you'll have to prove it by putting* $ax_1(t) + bx_2(t)$ *through it and verifying. Not time invariant: try inputs* $u(t)$ *and* $u(t-1)$ *as counterexample. Not causal, since* $y(-2)$ *depends on* $x(-1)$.

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3. *The output is*

$$y(t) = h(t) * x(t) = [\delta(t) - 4e^{-4t}u(t)] * p_2(t - 1/4) = p_2(t - 1/4) - 4y_1(t - 1/4)$$

with $y_1(t) = e^{-4t}u(t) * p_2(t)$. *This result can be found directly, or using the derivative property for convolution on* $p_2(t)$.

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4. *Evidently* $y(t) = 5x(t + 1/2) = \sum_{k=-\infty}^{\infty} 5c_k e^{jk\pi(t+1/2)} = \sum_{k=-\infty}^{\infty} 5c_k e^{jk\pi/2} e^{jk\pi t}$, *which is in the form of a Fourier series with coefficients* $d_k = 5c_k e^{jk\pi/2}$. *The Fourier transform for* $x(t)$ *is* $\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty} c_k 2\pi\delta(\omega - k\pi)$.

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5. *One way is to find Fourier transform of* $e^{-t}u(t)$ *and of* $\sin(2\pi t)$, *and to convolve in the frequency domain to give*

$$Y(\omega) = \frac{1}{2j}\left[\frac{1}{1 + j(\omega - 2\pi)} - \frac{1}{1 + j(\omega + 2\pi)}\right].$$

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6. *From tables we can find that* $|X(\omega)|^2 = 4/9 p_{6\pi}(\omega)$, *so the total energy is*

$E = 1/(2\pi) \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 4/3$ *joules.*

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7. *The transfer function* $H(\omega)$ *can be obtained from* $h(t)$*. From the definition of frequency response, the output when the input is* $x(t) = 3/2e^{j5t} + 3/2e^{-j5t}$ *is simply* $y(t) = 3/2H(5)e^{j5t} + 3/2H(-5)e^{-j5t}$*. Noting that* $H(-5) = H^*(5)$ *this can be simplified to* $y(t) = \frac{3}{\sqrt{100+25}} \cos(5t - \arctan(5/10))$.
