EEE235F MOCK EXAM
SIGNALS AND SYSTEMS I

University of Cape Town
Department of Electrical Engineering

June 2005

2 hours

Information

- The exam is closed-book.
- There are *eight* questions totalling 80 marks. You must answer all of them.
- The last page of this exam paper contains an information sheet with standard Fourier transforms and transform properties.
- You have 2 hours.

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1. The input $x(t)$ and the impulse response $h(t)$ of a continuous time LTI system are given by

$$x(t) = u(t) \qquad \text{and} \qquad h(t) = e^{-\alpha t}u(t), \quad \alpha > 0.$$

   (a) Compute the output $y(t)$ using $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$.

   (b) Compute the output $y(t)$ using $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$.

(10 marks)

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2. Let $x(t)$ and $v(t)$ be the signals

$$x(t) = u(t) \qquad \text{and} \qquad v(t) = e^{-\alpha t}u(t), \quad \alpha > 0.$$

   Sketch the following:

   (a) $x(t-\lambda)v(\lambda)$ for $t = 1$, $t = \frac{1}{2}$, and $t = 0$.

   (b) Hence find $w(0)$, $w(\frac{1}{2})$ and $w(1)$ if $w(t) = x(t) * v(t)$.

   (c) Also sketch $x(t-\lambda)\delta(\lambda)$ for $t = -\frac{1}{2}$ and $t = \frac{1}{2}$.

(10 marks)

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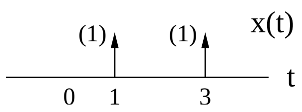
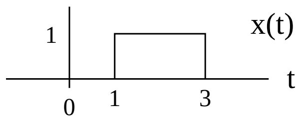
3. Consider a LTI system with *step* response

$$g(t) = e^{-t}u(t).$$

   (a) Determine and sketch the response to



   (b) Determine and sketch the response to



(10 marks)

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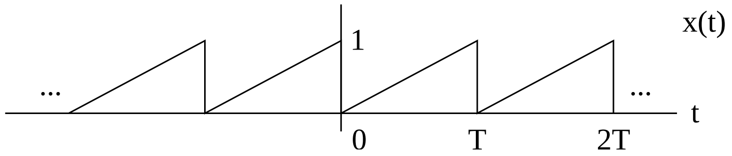
4. Find the trigonometric form with real coefficients of the Fourier series for the following signal:



(10 marks)

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5. (a) Show that the Fourier transform of $e^{-|t|}$ is $2/(1+\omega^2)$, and find the Fourier transform of the following signals:

$$x_1(t) = e^{-2|t|} \qquad x_2(t) = e^{-2|t|}\sin(t) \qquad x_3(t) = te^{-2|t|}.$$

(b) (4 marks) Use duality to find the Fourier transform of the signal

$$x(t) = \frac{1}{(1+t^2)}.$$

(10 marks)

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6. Find the transfer function of the system determined by the input/output relationship

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = \frac{dx}{dt} + 2x,$$

and determine its impulse response.

(10 marks)

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7. The signal $g(t)$ has the Fourier transform

$$G(\omega) = \frac{j\omega}{-\omega^2 + 5j\omega + 6}.$$

Find the Fourier transform of

(a) $g(2t)$

(b) $g(3t - 6)$

(c) $g(-t)$

(d) $\frac{dx(t)}{dt}$

(e) $e^{-j100t}g(t)$.

(10 marks)

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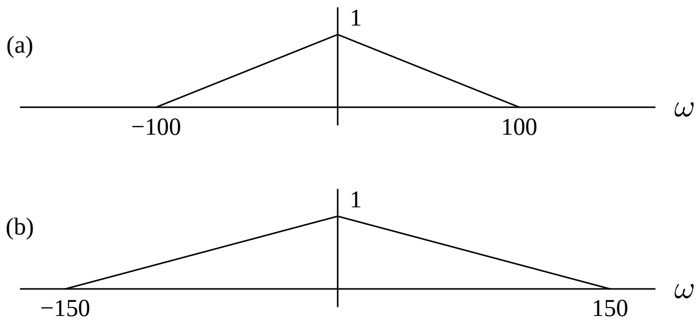
8. The signals with the frequency spectra shown below in (a) and (b) are sampled using an ideal sampler with $\omega_s = 200\text{rad}/s$. Sketch the frequency spectra of the sampled signals.



(10 marks)

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Multiplication by complex exponential	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt \leftrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t)$	$X(\omega)$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t - c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \qquad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\left(1 - \frac{2\|t\|}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2} \text{sinc}^2 \left(\frac{\tau\omega}{4\pi}\right)$