EEE235F EXAM
SIGNALS AND SYSTEMS I

HINTS: June 2005

1. *Fundamental. Part (e) is a bit tricky:* τ *is the variable and* t *a constant in this case, so the shift is to the right by* t*. Remember the two delta functions in (g).*

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2. *In the first part we are looking for smallest value of* T *such that* $\cos^2(t) = \cos^2(t+T)$*. Beware:* $T = 2\pi$ *is obviously a solution, but double angle formulae show that* $T = \pi$ *also works (plot* $\cos^2(t)$ *to see why). The signal in the second part is not periodic (plot it).*

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3. *Plotting* $h(t)$ *is simple. The step response of the system is the output when the input is the unit step:* $g(t) = h(t) * u(t)$ *(find directly using graphical convolution in the time domain). The system is causal because* $h(t)$ *is zero for* $t < 0$ *(convince yourself why by examining the convolution formula under this condition).*

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4. *The signal is symmetric and* $x(t) = e^{-at}$ *for* $t > 0$*, so* $x(t) = x_1(t) + x_1(-t)$ *with* $x_1(t) = e^{-at}u(t)$*. Finding* $X_1(\omega)$ *and noting that* $X(\omega) = X_1(\omega) + X(-\omega)$ *gives the result* $X(\omega) = 2a/(\omega^2 + a^2)$*.*

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5. *Since* $x(t) = \sin^2(t) = (1/(2j)(e^{jt} - e^{-jt}))^2 = 1/4e^{j2t} - 1/2 + 1/4e^{j2t}$*, the Fourier series follows directly:* $c_{-1} = c_1 = 1/4$ *and* $c_0 = 1/2$ *with* $\omega_0 = 2$*. Also, since* $x(t) = 1/2 - 1/2\cos(2t)$ *the nonzero coefficients of the trigonometric Fourier series are* $a_0 = 1/2$ *and* $a_1 = -1/2$*, with the same fundamental frequency.*

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6. *Taking the Fourier transform of the given expression and rearranging gives* $H(\omega) = 1/(2 + j\omega)$*. The response to the given* $x(t)$ *can be found by multiplying with*

$X(\omega)$ *in the frequency domain, doing a partial fraction expansion on the product, and inverse transforming to give* $y(t) = e^{-t}u(t) - e^{-2t}u(t)$. *The impulse response is the inverse transform of* $H(\omega)$, *namely* $h(t) = e^{-2t}u(t)$.

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7. *Using the multiplication by cosine*$\cos(\omega_0 t)$ *property on the pair.*

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