# EEE235F EXAM
# SIGNALS AND SYSTEMS I


University of Cape Town
Department of Electrical Engineering


June 2005
2 hours

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**Information**

- The exam is closed-book.
- There are *eight* questions totalling 90 marks. You must answer all of them.
- The last two pages of this exam paper contain information sheets with standard Fourier transforms, transform properties, and some trigonometric identities.
- You have 2 hours.

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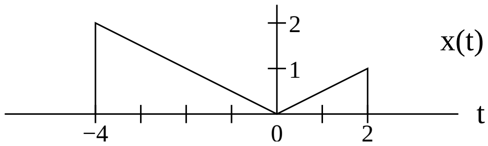
1. Consider the signal $x(t)$ below:




Sketch the following:

(a) $y(t) = 2 - 2x(t)$

(b) $y(t) = x(-t+1)$

(c) $y(t) = x(t+2)$

(d) $y(t) = x(t/2)$

(e) $y(\tau) = x(\tau - t)$

(f) $y(t) = x^2(t)$

(g) $y(t) = \frac{dx(t)}{dt}$ (the generalised derivative).

(15 marks)

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2. Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = \cos^2(t)$

(b) $x(t) = \cos(2\pi t)u(t)$.

(10 marks)

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3. An LTI system has impulse response $h(t) = e^{-\alpha t}u(t)$ with $\alpha > 0$.

(a) Carefully sketch this impulse response for $\alpha = 2$.

(b) Use time-domain convolution to find the step response of the system.

(c) Is the system causal? Why?

(10 marks)

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4. Sketch the signal $x(t) = e^{-a|t|}$ for $a > 0$, and find its Fourier transform.

(10 marks)

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5. Given the signal $x(t) = \sin^2(t)$:

   (a) Find the complex exponential Fourier series representation of $x(t)$

   (b) Determine the trigonometric Fourier series representation of $x(t)$.

(10 marks)

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6. Consider the continuous-time LTI system described by

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

   (a) Using the Fourier transform, find the transfer function of the system

   (b) Find the output of the system for the input $x(t) = e^{-t}u(t)$

   (c) Find the impulse response of the system.

(15 marks)

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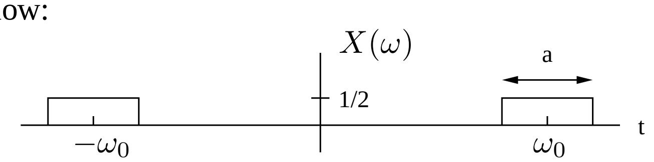
7. The Fourier transform of a signal $x(t)$ is given by

$$X(\omega) = \frac{1}{2}p_a(\omega - \omega_0) + \frac{1}{2}p_a(\omega + \omega_0),$$

which is shown below:




Find an expression for $x(t)$.

(10 marks)

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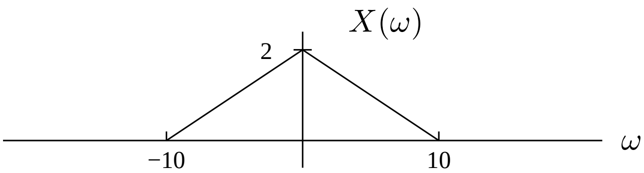
8. Consider the signal whose Fourier transform is shown below:




Let $x_s(t) = x(t)p(t)$ represent the sampled signal (where $p(t)$ is the impulse train with period $T$).

   (a) Find the Fourier series representation for the impulse train $p(t)$

   (b) Draw $X_s(\omega)$ for $T = \pi/15$ and $T = 2\pi/15$.

(10 marks)

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# INFORMATION SHEET

## Fourier transform properties

| Property | Transform Pair/Property |
|---|---|
| Linearity | $ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$ |
| Time shift | $x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$ |
| Time scaling | $x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$ |
| Time reversal | $x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$ |
| Multiplication by power of $t$ | $t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$ |
| Frequency shift | $x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real |
| Multiplication by $\cos(\omega_0 t)$ | $x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$ |
| Differentiation in time domain | $\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$ |
| Integration | $\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$ |
| Convolution in time domain | $x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$ |
| Multiplication in time domain | $x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$ |
| Parseval's theorem | $\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$ |
| Parseval's theorem (special case) | $\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$ |
| Duality | $X(t) \leftrightarrow 2\pi x(-\omega)$ |

## Common Fourier Transform Pairs

| $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$ | $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ |
|---|---|
| $1 \quad (-\infty < t < \infty)$ | $2\pi\delta(\omega)$ |
| $-0.5 + u(t)$ | $\frac{1}{j\omega}$ |
| $u(t)$ | $\pi\delta(\omega) + \frac{1}{j\omega}$ |
| $\delta(t)$ | $1$ |
| $\delta(t-c)$ | $e^{-j\omega c}$ ($c$ any real number) |
| $e^{-bt}u(t)$ | $\frac{1}{j\omega + b} \quad (b > 0)$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0)$ ($\omega_0$ any real number) |
| $p_\tau(t)$ | $\tau \text{sinc} \frac{\tau\omega}{2\pi}$ |
| $\tau \text{sinc} \frac{\tau t}{2\pi}$ | $2\pi p_\tau(\omega)$ |
| $\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$ | $\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$ |
| $\frac{\tau}{2}\text{sinc}^2 \frac{\tau t}{4\pi}$ | $2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$ |
| $\cos(\omega_0 t + \theta)$ | $\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$ |
| $\sin(\omega_0 t + \theta)$ | $j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$ |

## Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$