EEE2047S: Signals and Systems I

Class Test

6 October 2021

Please do not put your name or student number anywhere on this script.

Peoplesoft ID:

Information

- The test is closed-book.
- This test has *four* questions, totaling 20 marks.
- There is an information sheet attached at the end of this paper.
- Answer *all* the questions.
- You have 60 minutes.

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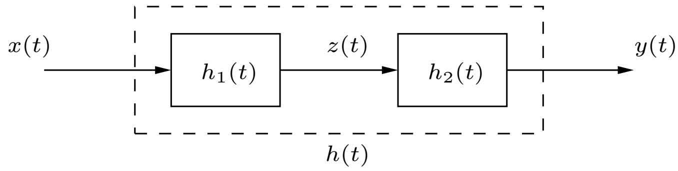
1. (5 marks) A modulator used in AM radio transmission has the input-output relationship

$$y(t) = x(t)\cos(\omega_c t),$$

where ω_c is a known and fixed carrier frequency. Determine whether the modulator system is (a) linear, (b) time invariant, and (c) causal.

2. (5 marks) Consider two systems connected as shown below,



where the impulse responses of the two systems are $h_1(t) = u(t)$ and

$$h_2(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find and sketch the effective impulse response $h(t)$ linking the input $x(t)$ to the output $y(t)$.

3. (5 marks) The signal $f(t) = -2\cos(\omega_0 t) + 2\sqrt{3}\sin(\omega_0 t + \pi/3)$ can be written in the form

$$f(t) = ce^{j\omega_0 t} + c^* e^{-j\omega_0 t},$$

where c^* is the conjugate of c. Find the value of c in this representation.

Recall Euler's formula $e^{j\theta} = \cos(\theta) + j\sin(\theta)$.

4. (5 marks)

(a) Find the Fourier transform of the triangular pulse signal defined as follows:

$$x_1(t) = \begin{cases} 1+t & -1 \leq t \leq 0 \\ 1-t & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Find and sketch the Fourier transform of the signal

$$x_2(t) = 2\text{sinc}^2(3t).$$

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \Longleftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \Longleftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \Longleftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \Longleftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \Longleftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \Longleftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \Longleftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \Longleftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \Longleftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \Longleftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \Longleftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \Longleftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \qquad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \mathrm{sinc}\frac{\tau\omega}{2\pi}$
$\tau \mathrm{sinc}\frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2}\mathrm{sinc}^2\frac{\tau t}{4\pi}$	$2\pi\left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$
$\sum_{n=-\infty}^{\infty}\delta(t - nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - k\frac{2\pi}{T})$

with $p_\tau(t) = u(t + \tau/2) - u(t - \tau/2)$ and $\mathrm{sinc}(\lambda) = \sin(\pi\lambda)/(\pi\lambda)$.

Laplace transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \Longleftrightarrow aX(s) + bV(s)$
Time shift	$x(t-a)u(t-a) \Longleftrightarrow e^{-as}X(s) \quad a \geq 0$
Time scaling	$x(at) \Longleftrightarrow \frac{1}{a}X(\frac{s}{a}) \quad a > 0$
Frequency differentiation	$t^n x(t) \Longleftrightarrow (-1)^n X^{(n)}(s)$
Frequency shift	$e^{at}x(t) \Longleftrightarrow X(s-a)$
Differentiation	$x'(t) \Longleftrightarrow sX(s) - x(0^-)$
	$x''(t) \Longleftrightarrow s^2X(s) - sx(0^-) - x'(0^-)$
	$x^{(n)}(t) \Longleftrightarrow s^nX(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$
Integration	$\int_{0^-}^{t} x(\lambda)d\lambda \Longleftrightarrow \frac{1}{s}X(s)$
	$\int_{-\infty}^{t} x(\lambda)d\lambda \Longleftrightarrow \frac{1}{s}X(s) + \frac{1}{s}\int_{-\infty}^{0^-} x(\lambda)d\lambda$
Time convolution	$x(t) * v(t) \Longleftrightarrow X(s)V(s)$
Frequency convolution	$x(t)v(t) \Longleftrightarrow \frac{1}{2\pi j}X(s) * V(s)$

Initial value: $f(0^+) = \lim_{s\to\infty} sF(s)$

Final value: $f(\infty) = \lim_{s\to 0} sF(s)$ with all poles in left-hand plane

Common Unilateral Laplace Transform Pairs

$x(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$	$X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$
$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$
$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
$\cos(bt)u(t)$	$\frac{s}{s^2+b^2}$
$\sin(bt)u(t)$	$\frac{b}{s^2+b^2}$
$e^{-at}\cos(bt)u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
$e^{-at}\sin(bt)u(t)$	$\frac{b}{(s+a)^2+b^2}$
$re^{-at}\cos(bt+\theta)u(t)$	$\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$
$re^{-at}\cos(bt+\theta)u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb} + \frac{0.5re^{-j\theta}}{s+a+jb}$
$re^{-at}\cos(bt+\theta)u(t)$	$\frac{As+B}{s^2+2as+c}$
	$r = \sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}$, $b = \sqrt{c-a^2}$, $\theta = \tan^{-1}\frac{Aa-b}{A\sqrt{c-a^2}}$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta) \qquad \sin^2(\theta) + \cos^2(\theta) = 1$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$