EEE2047S: Signals and Systems I

Class Test 1

27 August 2018

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totaling 20 marks.
- There is an information sheet attached at the end of this paper.
- Answer *all* the questions.
- You have 60 minutes.

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1. (5 marks) Evaluate the following integrals:
   (a) $\int_{-\infty}^{\infty} \cos(5\pi t)\delta(t)dt$
   (b) $\int_{5}^{\infty} \cos(5\pi t)\delta(t)dt$
   (c) $\int_{-\infty}^{\infty} \cos(5\pi(t-\lambda))\delta(t)dt$
   (d) $\int_{-\infty}^{\infty} \cos(5\pi(t-\lambda))\delta(\lambda)d\lambda$
   (e) $\int_{0}^{\infty} \cos(5\pi t)\delta(t-\lambda)d\lambda$

2. (5 marks) Consider a system where the input $x(t)$ and output $y(t)$ are related by the following:

$$y(t) = x(t^2).$$

(a) Is the system linear? Justify.

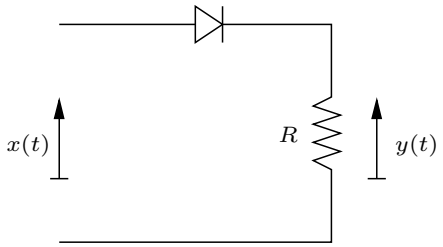
(b) Is the system causal? Justify.

(c) Find and plot the output for the two cases where $x(t) = u(t)$ and $x(t) = u(t-1)$.

(d) Is the system time invariant? Justify.

3. (5 marks) Suppose we define the right-sided ramp signal as $r(t) = tu(t)$, where $u(t)$ is the unit step. Then:
   (a) Sketch $r(t)$.
   (b) Show that $u(t) * u(t+1) = r(t+1)$, where $u(t)$ is the unit step.
   (c) Use the previous result to find $y(t) = u(t) * p_1(t - 1/2)$, where $p_1(t)$ is a unit pulse centered on the origin with a total width of one.

4. (5 marks) The half-wave rectifier circuit below is driven by a voltage signal $x(t)$, and the output voltage across the resistor is $y(t)$:



Assume an ideal diode with zero forward voltage drop.

(a) Sketch the output $y(t)$ when the input is $x(t) = \sin(t)$.

(b) What is the fundamental period of $y(t)$?

(c) What is the average power of $y(t)$?

(d) The output can be written in the form

$$y(t) = \sum_{k=-\infty}^{\infty} c_k e^{jkt}.$$

Find the value of c_1 in this representation, expressed in polar form.