

# EEE2047S: Signals and Systems I

## Class Test 1

15 September 2017

## SOLUTIONS

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Name:

Student number:

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### Information

- The test is closed-book.
  - This test has *four* questions, totaling 20 marks.
  - There is an information sheet attached at the end of this paper.
  - Answer *all* the questions.
  - You have 45 minutes.
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1. (5 marks) Consider the system described by the input-output relationship

$$y(t) = \int_{-\infty}^t x(\tau - 2) d\tau,$$

with  $y(t)$  the output when  $x(t)$  is the input.

- (a) Suppose you want to calculate  $y(0)$ . For what set of values of  $t$  do you need to know  $x(t)$ ?
- (b) Is this system causal? Why or why not?
- (c) You don't need to show it, but the system is linear and time invariant. Find and sketch the impulse response  $h(t)$ , which is the output when the input is  $x(t) = \delta(t)$ .

- (a) A change of variables  $p = \tau - 2$  yields

$$y(t) = \int_{-\infty}^{t-2} x(p) dp.$$

Thus to calculate  $y(0) = \int_{-\infty}^{-2} x(p) dp$  we need to know  $x(t)$  over the range  $-\infty < t \leq -2$ .

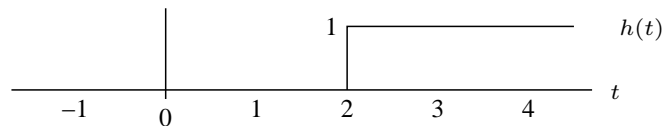
- (b) From the above we see that to calculate  $y(t)$  at  $t = t_0$  we need to know  $x(t)$  over the range  $-\infty < t \leq t_0 - 2$ . These values are all in the past, so the system is causal.
- (c) The impulse response satisfies

$$h(t) = \int_{-\infty}^t \delta(\tau - 2) d\tau = \int_{-\infty}^{t-2} \delta(p) dp.$$

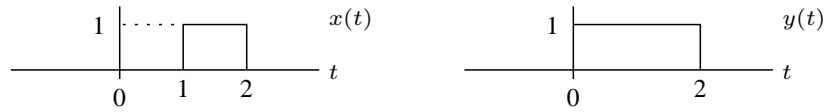
The range of this integration only includes the delta function when the upper limit satisfies  $t - 2 > 0$ , in which case the integral is one, otherwise it is zero. Thus

$$h(t) = \begin{cases} 1 & t > 2 \\ 0 & \text{otherwise,} \end{cases}$$

or  $h(t) = u(t - 2)$ . Plot follows:

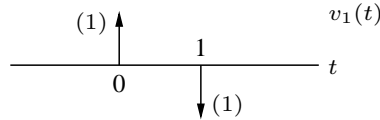


2. (5 marks) When the input to a LTI system is  $x(t)$  below then the output is  $y(t)$ :

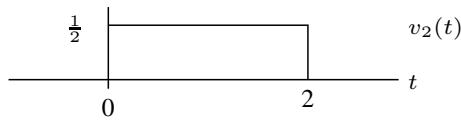


Find and plot the following:

- (a) The output when the input is the signal  $v_1(t)$  below:

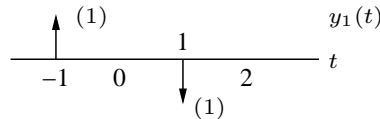


- (b) The output when the input is the signal  $v_2(t)$  below:

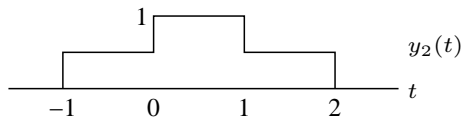


The system is LTI and it therefore has an impulse response  $h(t)$ . We know that  $y(t) = h(t) * x(t)$  for the signals given.

- (a) Observing that  $\dot{x}(t) = v_1(t-1)$  we have  $\dot{x}(t+1) = v_1(t)$ . Since  $\dot{y}(t+1) = h(t) * \dot{x}(t+1) = h(t) * v_1(t)$  we note that the required output is  $y_1(t) = \dot{y}(t+1)$ , plotted below:



- (b) We note that  $v_2(t)$  can be constructed from  $x(t)$  using shifting, scaling, and addition according to  $v_2(t) = 1/2x(t) + 1/2x(t+1)$ . Since the response to  $x(t)$  is  $y(t)$ , the response to  $v_2(t)$  will be  $y_2(t) = 1/2y(t) + 1/2y(t+1)$ , plotted below:



3. (5 marks) This question has two independent parts:

(a) The signal

$$x(t) = \sin\left(\frac{7\pi t}{3} + \frac{\pi}{3}\right)$$

can be written in the form  $x(t) = ce^{7\pi t/3} + c^*e^{-7\pi t/3}$ , where  $c^*$  is the complex conjugate of  $c$ . Find  $c$  in polar form.

(b) The signal  $w(t) = \delta(t-3)\frac{4-jt^2}{2t}$  can be written in the form  $w(t) = k\delta(t-3)$  for some complex number  $k$ . Find  $k$  in rectangular form.

(a) We can write the signal as follows:

$$x(t) = \frac{1}{2j} \left( e^{j(7\pi t/3 + \pi/3)} - e^{-j(7\pi t/3 + \pi/3)} \right) = \frac{1}{2j} e^{j\pi/3} e^{j7\pi t/3} - \frac{1}{2j} e^{-j\pi/3} e^{-j7\pi t/3},$$

so  $c = \frac{1}{2j} e^{j\pi/3}$ . Since  $j = e^{j\pi/2}$  this can be written as required:

$$c = \frac{1}{2} e^{j\pi/3} e^{-j\pi/2} = \frac{1}{2} e^{-j\pi/6}$$

(b) Using the sifting property gives

$$w(t) = \delta(t-3) \frac{4-jt^2}{2t} = \delta(t-3) \frac{4-j(3)^2}{2(3)},$$

so

$$k = \frac{4-j(3)^2}{2(3)} = \frac{4}{6} - j\frac{9}{6} = \frac{2}{3} - j\frac{3}{2}.$$

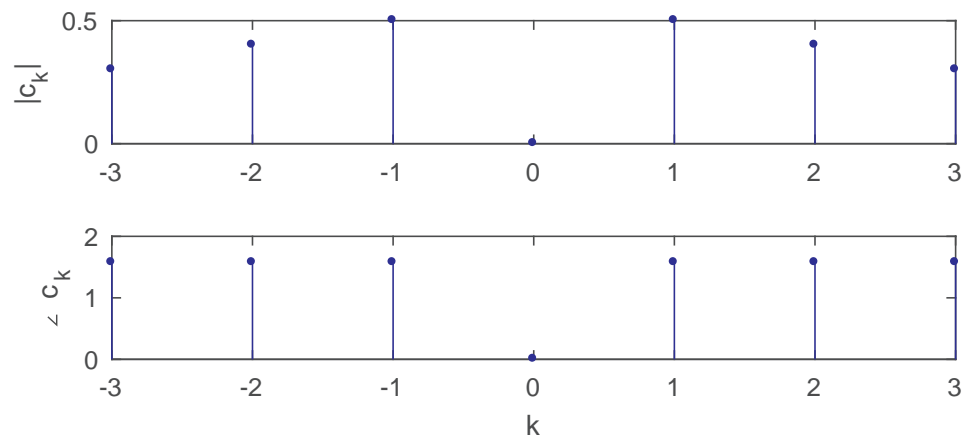
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4. (5 marks) Let  $x(t)$  be a periodic function, with fundamental frequency  $\omega_0 = 4$  rad/s and with Fourier series in exponential form

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{j|k|}{1+k^2} e^{4jkt} = \sum_{k=-\infty}^{\infty} c_k e^{4jkt}.$$

- What is the fundamental period of the signal?
- Plot the magnitude and phase of the Fourier series coefficients  $c_k$  over the range  $k = -3$  to  $k = 3$ .
- What is the average value of the signal over one period?
- How much signal power is contained in the second harmonic?

- Since  $T = 2\pi/\omega_0$  we have  $T = 2\pi/4 = \pi/2$  seconds.
- Note that  $c_k = |k|/(1+k^2)e^{j\pi/2}$ . Since  $|k|/(1+k^2) \geq 0$  we have  $|c_k| = |k|/(1+k^2)$  and  $\angle c_k = \pi/2$ . Plots follow:



- The average value is  $c_0 = 0$ .
- The total power in the second harmonic is

$$\begin{aligned} P_2 &= |c_2|^2 + |c_{-2}|^2 = (|2|/(1+2^2))^2 + (|-2|/(1+(-2)^2))^2 \\ &= (2/(1+4))^2 + (2/(1+4))^2 = 2(4/25) = 8/25 \text{ Watts.} \end{aligned}$$


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