EEE2047S: Signals and Systems I

Class Test 1

15 September 2017

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totaling 20 marks.
- There is an information sheet attached at the end of this paper.
- Answer *all* the questions.
- You have 45 minutes.

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1. (5 marks) Consider the system described by the input-output relationship

$$y(t) = \int_{-\infty}^{t} x(\tau - 2) d\tau,$$

with $y(t)$ the output when $x(t)$ is the input.

(a) Suppose you want to calculate $y(0)$. For what set of values of t do you need to know $x(t)$?

(b) Is this system causal? Why or why not?

(c) You don't need to show it, but the system is linear and time invariant. Find and sketch the impulse response $h(t)$, which is the output when the input is $x(t) = \delta(t)$.

(a) A change of variables $p = \tau - 2$ yields

$$y(t) = \int_{-\infty}^{t-2} x(p) dp.$$

Thus to calculate $y(0) = \int_{-\infty}^{-2} x(p) dp$ we need to know $x(t)$ over the range $-\infty < t \leq 2$.

(b) From the above we see that to calculate $y(t)$ at $t = t_0$ we need to know $x(t)$ over the range $-\infty < t \leq t_0 - 2$. These values are all in the past, so the system is causal.

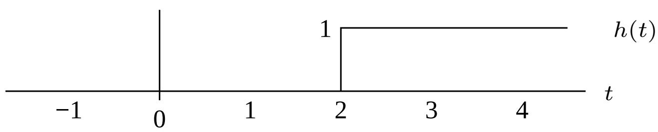
(c) The impulse response satisfies

$$h(t) = \int_{-\infty}^{t} \delta(\tau - 2) d\tau = \int_{-\infty}^{t-2} \delta(p) dp.$$

The range of this integration only includes the delta function when the upper limit satisfies $t - 2 > 0$, in which case the integral is one, otherwise it is zero. Thus

$$h(t) = \begin{cases} 1 & t > 2 \\ 0 & \text{otherwise,} \end{cases}$$

or $h(t) = u(t - 2)$. Plot follows:



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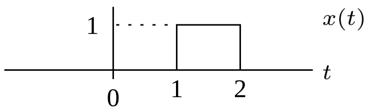
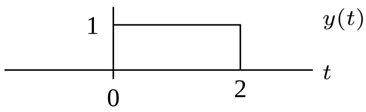
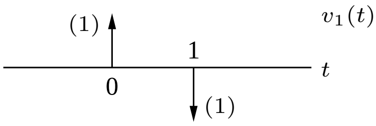
2. (5 marks) When the input to a LTI system is $x(t)$ below then the output is $y(t)$:



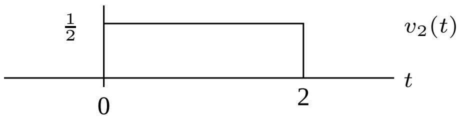


Find and plot the following:

(a) The output when the input is the signal $v_1(t)$ below:

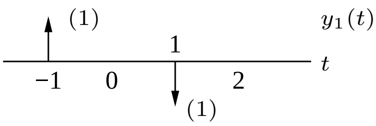


(b) The output when the input is the signal $v_2(t)$ below:

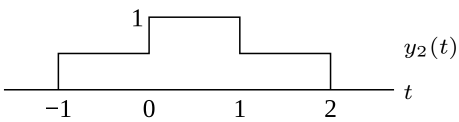


The system is LTI and it therefore has an impulse response $h(t)$. We know that $y(t) = h(t) * x(t)$ for the signals given.

(a) Observing that $\dot{x}(t) = v_1(t-1)$ we have $\dot{x}(t+1) = v_1(t)$. Since $\dot{y}(t+1) = h(t) * \dot{x}(t+1) = h(t) * v_1(t)$ we note that the required output is $y_1(t) = \dot{y}(t+1)$, plotted below:



(b) We note that $v_2(t)$ can be constructed from $x(t)$ using shifting, scaling, and addition according to $v_2(t) = 1/2x(t) + 1/2x(t+1)$. Since the response to $x(t)$ is $y(t)$, the response to $v_2(t)$ will be $y_2(t) = 1/2y(t) + 1/2y(t+1)$, plotted below:



3. (5 marks) This question has two independent parts:

(a) The signal

$$x(t) = \sin\left(\frac{7\pi t}{3} + \frac{\pi}{3}\right)$$

can be written in the form $x(t) = ce^{7\pi t/3} + c^* e^{-7\pi t/3}$, where c^* is the complex conjugate of c. Find c in polar form.

(b) The signal $w(t) = \delta(t-3)\frac{4-jt^2}{2t}$ can be written in the form $w(t) = k\delta(t-3)$ for some complex number k. Find k in rectangular form.

(a) We can write the signal as follows:

$$x(t) = \frac{1}{2j}\left(e^{j(7\pi t/3+\pi/3)} - e^{-j(7\pi t/3+\pi/3)}\right) = \frac{1}{2j}e^{j\pi/3}e^{j7\pi t/3} - \frac{1}{2j}e^{-j\pi/3}e^{-j7\pi t/3},$$

so $c = \frac{1}{2j}e^{j\pi/3}$. Since $j = e^{j\pi/2}$ this can be written as required:

$$c = \frac{1}{2}e^{j\pi/3}e^{-j\pi/2} = \frac{1}{2}e^{-j\pi/6}$$

(b) Using the sifting property gives

$$w(t) = \delta(t-3)\frac{4-jt^2}{2t} = \delta(t-3)\frac{4-j(3)^2}{2(3)},$$

so

$$k = \frac{4-j(3)^2}{2(3)} = \frac{4}{6} - j\frac{9}{6} = \frac{2}{3} - j\frac{3}{2}.$$

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4. (5 marks) Let $x(t)$ be a periodic function, with fundamental frequency $\omega_0 = 4$ rad/s and with Fourier series in exponential form

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{j|k|}{1+k^2} e^{4jkt} = \sum_{k=-\infty}^{\infty} c_k e^{4jkt}.$$

(a) What is the fundamental period of the signal?

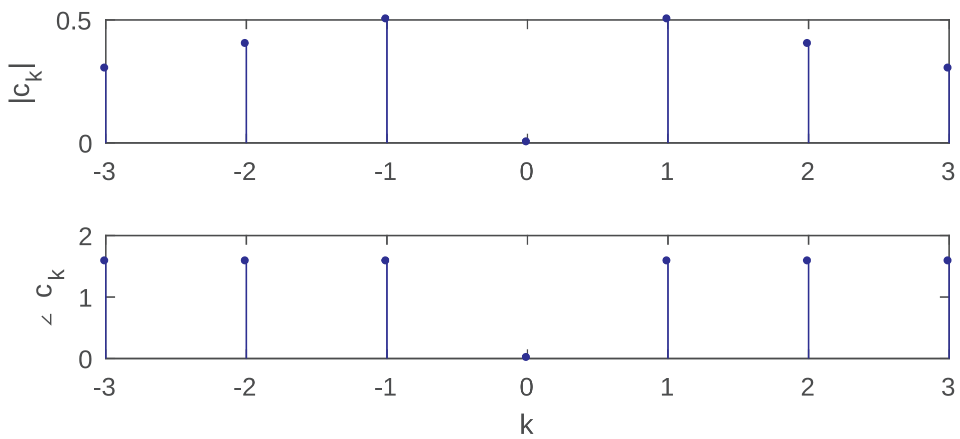
(b) Plot the magnitude and phase of the Fourier series coefficients c_k over the range $k = -3$ to $k = 3$.

(c) What is the average value of the signal over one period?

(d) How much signal power is contained in the second harmonic?

(a) Since $T = 2\pi/\omega_0$ we have $T = 2\pi/4 = \pi/2$ seconds.

(b) Note that $c_k = |k|/(1+k^2)e^{j\pi/2}$. Since $|k|/(1+k^2) \geq 0$ we have $|c_k| = |k|/(1+k^2)$ and $\angle c_k = \pi/2$. Plots follow:



(c) The average value is $c_0 = 0$.

(d) The total power in the second harmonic is

$$P_2 = |c_2|^2 + |c_{-2}|^2 = (|2|/(1+2^2))^2 + (|-2|/(1+(-2)^2))^2$$
$$= (2/(1+4))^2 + (2/(1+4))^2 = 2(4/25) = 8/25 \text{ Watts.}$$