EEE2035F: Signals and Systems I

Class Test 1

14 March 2016

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totaling 20 marks.
- There is an information sheet attached at the end of this paper.
- Answer *all* the questions.
- You have 45 minutes.

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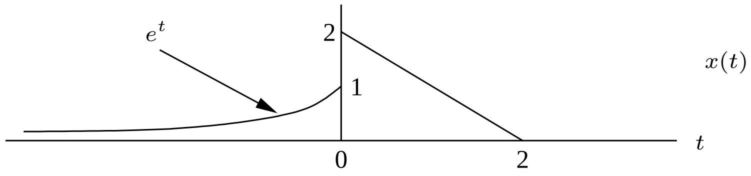
1. (5 marks) Consider the signal $x(t)$ below:



Sketch the following:

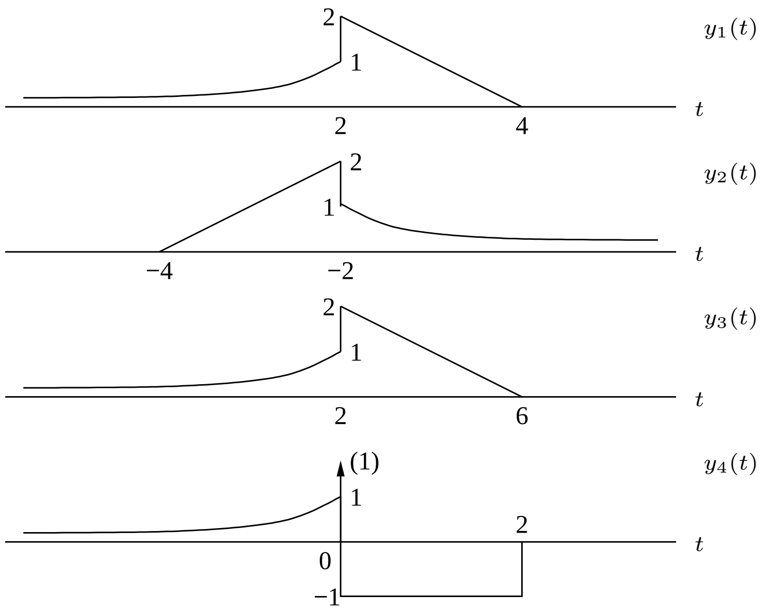
(a) $y_1(t) = x(t-2)$

(b) $y_2(t) = x(-t-2)$

(c) $y_3(t) = x(t/2-1)$

(d) $y_4(t) = \frac{d}{dt}x(t)$.

Plots as follows:



2. (5 marks) A system with input $x(t)$ and output $y(t)$ is governed by the input-output relationship

$$y(t) = \int_0^\infty e^{-\tau} x(t-\tau) d\tau.$$

(a) Show that the system is time invariant.

(b) Assuming that the system in linear and time invariant, find its impulse response $h(t)$.

(a) Inputs $x_1(t)$ and $x_2(t)$ will produce the following input-output pairs:

$$\begin{aligned} x_1(t) &\longrightarrow y_1(t) = \int_0^\infty e^{-\tau} x_1(t-\tau) d\tau \\ x_2(t) &\longrightarrow y_2(t) = \int_0^\infty e^{-\tau} x_2(t-\tau) d\tau. \end{aligned}$$

Suppose in particular that $x_2(t) = x_1(t-c)$. Then

$$y_2(t) = \int_0^\infty e^{-\tau} x_1(t-c-\tau) d\tau = y_1(t-c).$$

Thus the shifted input $x_2(t) = x_1(t-c)$ always results in the shifted output $y_2(t) = y_1(t-c)$ and the system is time invariant.

(b) The impulse response is the output when the input is the delta function at the origin, or $x(t) = \delta(t)$, so

$$h(t) = \int_0^\infty e^{-\tau} \delta(t-\tau) d\tau = \int_0^\infty e^{-t} \delta(t-\tau) d\tau = e^{-t} \int_0^\infty \delta(t-\tau) d\tau.$$

However

$$\int_0^\infty \delta(t-\tau) d\tau = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

so $h(t) = e^{-t} u(t)$.

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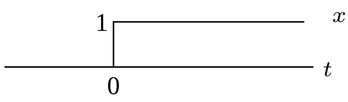
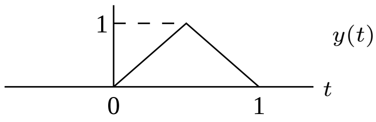
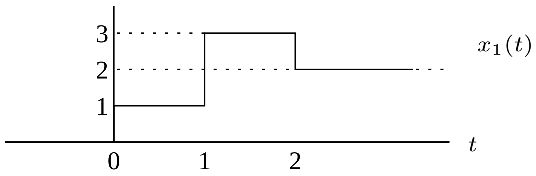
3. (5 marks) Suppose we have a linear time-invariant system for which the input $x(t)$ below produces the output $y(t)$:





Find the output $y_1(t)$ when the input is the signal $x_1(t)$:



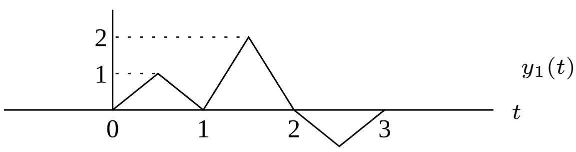
An input $x(t) = u(t)$ produces the output $y(t)$. However, we can write the given input as

$$x_1(t) = u(t) + 2u(t-1) - u(t-2).$$

By directly applying linearity and time invariance the output in this case will be

$$y_1(t) = y(t) + 2y(t-1) - y(t-2),$$

which looks as follows:



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4. (5 marks) Find and plot $y(t) = h(t) * x(t)$ when $h(t) = e^{-t}u(t)$ and $x(t) = u(t-1)$.

To simplify things we find $y_1(t) = h(t) * u(t)$. From time invariance (shifting property) we then know that $y_1(t-1) = h(t) * u(t-1) = y(t)$, the required output signal.

Using the derivative property $\dot{y}_1(t) = h(t) * \dot{u}(t) = h(t) * \delta(t) = h(t)$. We can therefore calculate

$$y_1(t) = \int_{-\infty}^{t} h(\lambda) d\lambda = \int_{-\infty}^{t} e^{-\lambda} u(\lambda) d\lambda.$$

If $t < 0$ then the integrand is always zero over the range of integration, so $y_1(t) = 0$. For $t \geq 0$ we have

$$y_1(t) = \int_0^t e^{-\lambda} d\lambda = \left[-e^{-\lambda} \right]_{\lambda=0}^{t} = 1 - e^{-t}.$$

Thus $y_1(t) = (1 - e^{-t})u(t)$ and $y(t) = (1 - e^{-(t-1)})u(t-1)$.
